Mobile Communications

Part VIII- Propagation Characteristics
Multi-path Propagation - Fading

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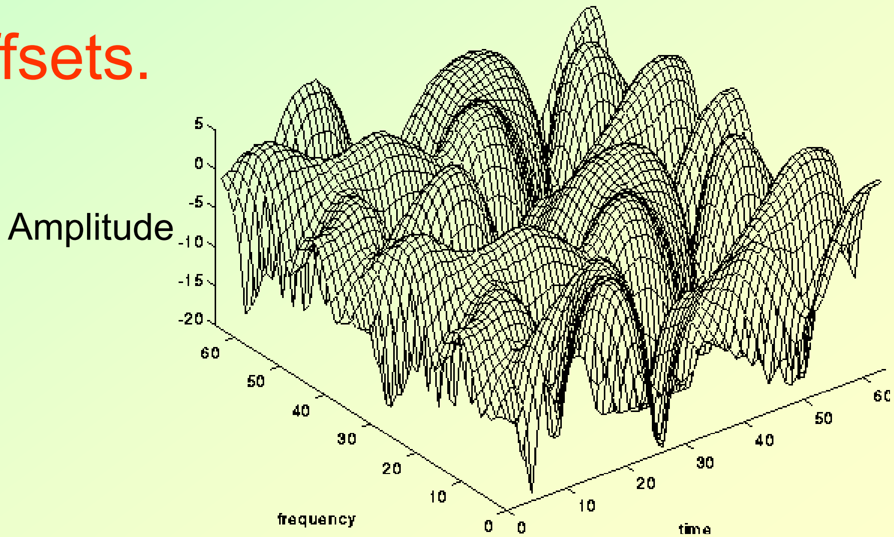
Contents

- Fading
- Doppler Shift
- Dispersion
- Summary

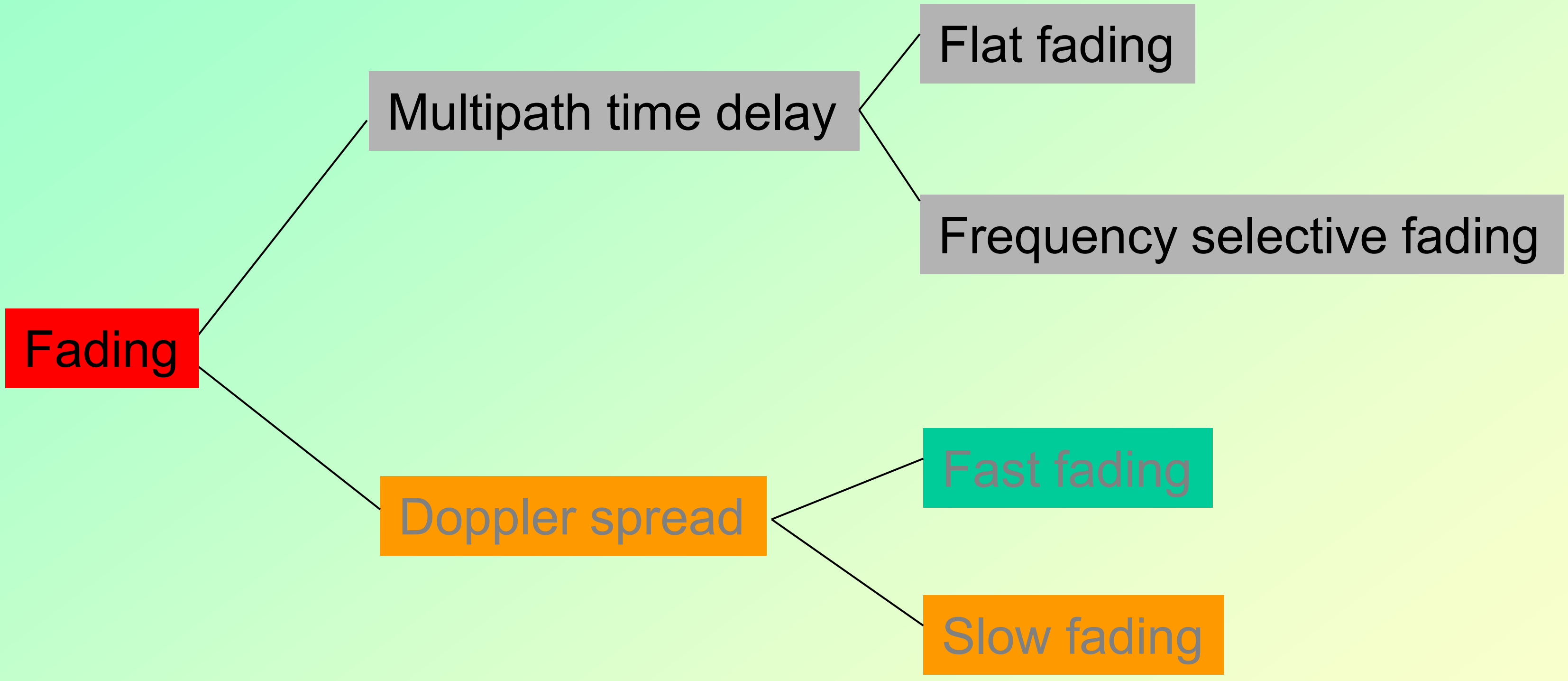
The Mobile Radio Propagation Channel

- A wireless channel exhibits severe fluctuations for small:
  - displacements of the antennas
  - variation of intensity
  - Small carrier frequency offsets.

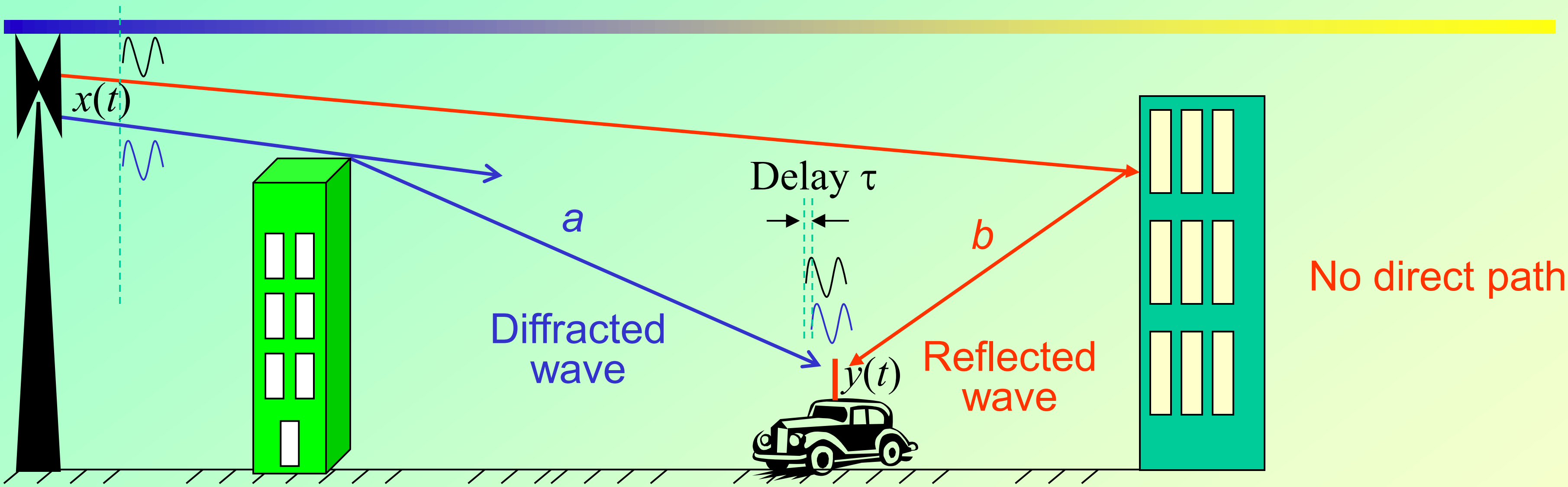


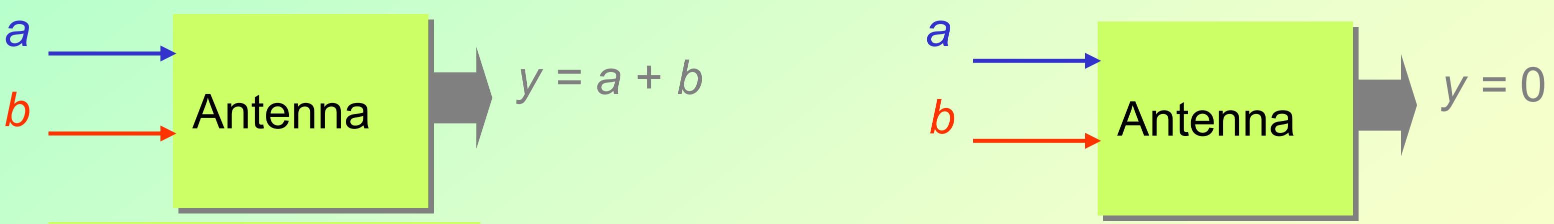
Channel amplitude in dB versus location (= time*velocity) and frequency

Fading – Small Scale



Fading - Multipath Propagation





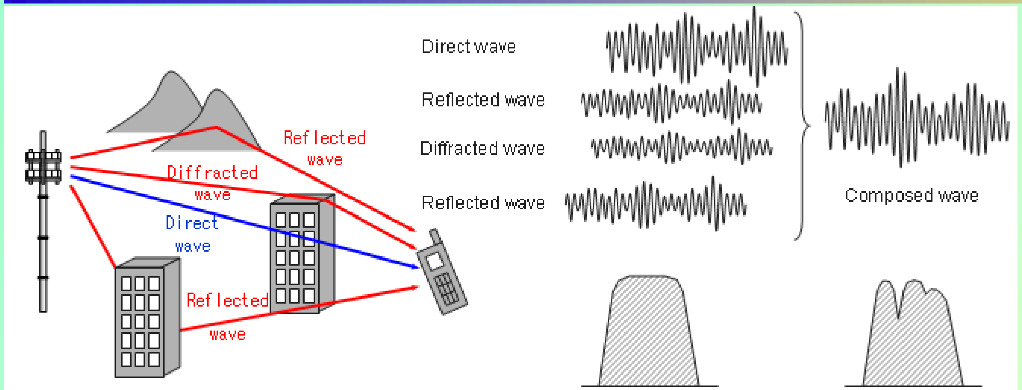
a & *b* are in phase

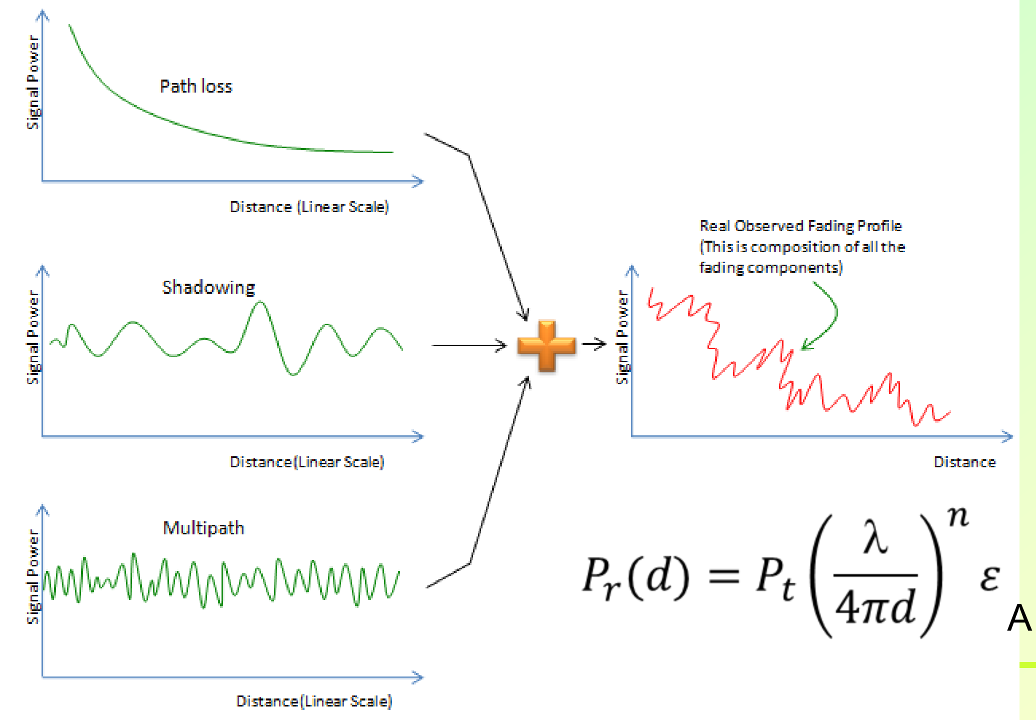
a & *b* are out of phase by π

Complete fading when

$2\pi\Delta d/\lambda = n\pi$, where Δd is the path difference = *b*-*a*

Channel Fading - Multipath Propagation - *contd.*





$$P_r(d) = P_t\left(\frac{\lambda}{4\pi d}\right)^n \varepsilon$$

A random variable

- For a stationary mobile unit with no direct path, the received signal can be expressed as a sum of delayed components or in terms of phasor notation:

Pulse train

$$S_r(t) = \sum_{i=1}^{N} a_i P(t - \tau_i)$$

A single frequency

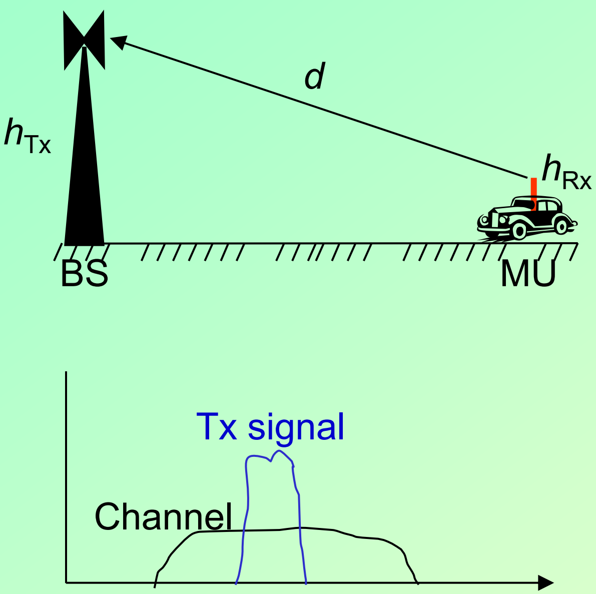
$$S_r(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_c + \phi_i)$$

Where:
a_i is the amplitude of the scattered signal,
$p(t)$ is the transmitted signal (pulse) shape,
τ_i is the time taken by the pulse to reach the receiver,
N is the number of different paths
f_c is the carrier frequency

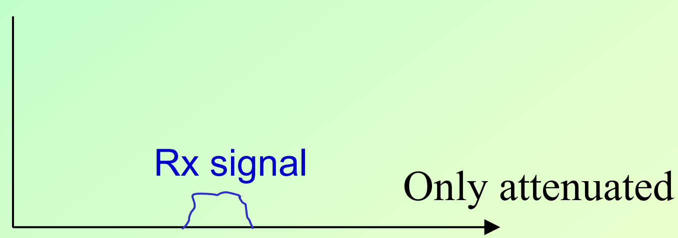
http://www.sharetechnote.com/html/Handbook_LTE_Fading.html

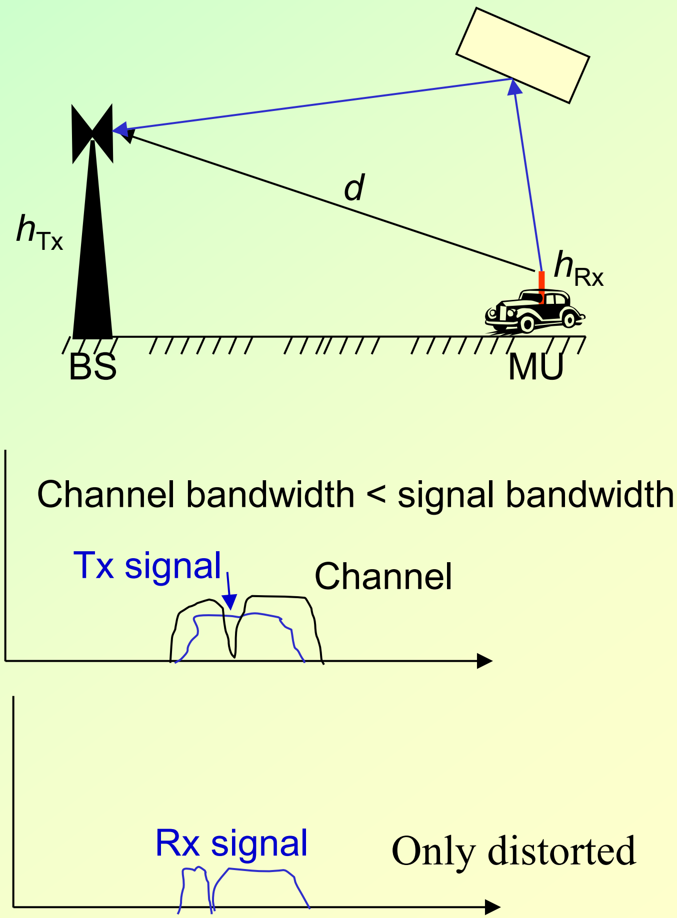
Channel Fading - Multipath Propagation - *contd*.

• Flat vs. Frequency selective fading



- In free space signal attenuation α $1/d^2$ (i.e., 20 dB/dec)
- In real case with reflection attenuation α $1/d^4$ (i.e., 40 dB/dec) given that $d >> \sqrt{[h_{Tx} h_{Rx}]}$

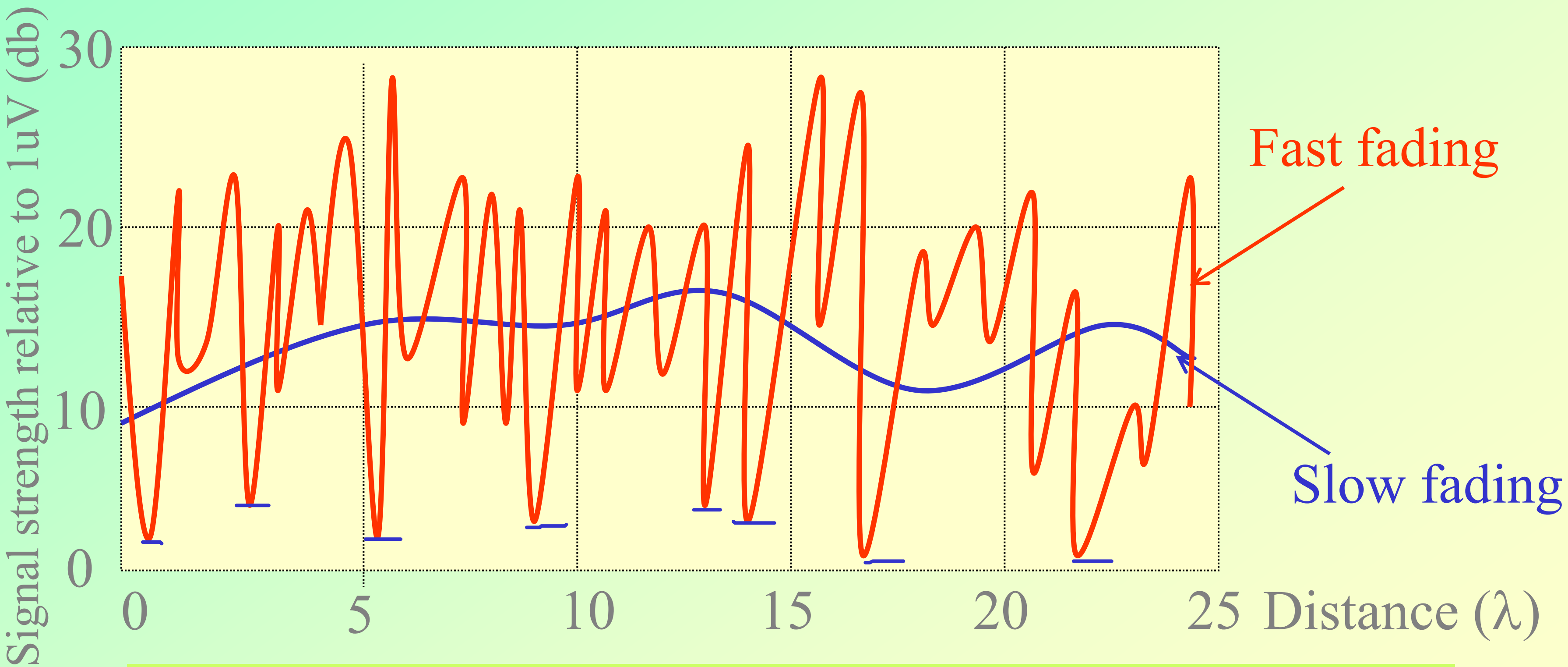




Frequency selective fading is the most common In mobile environments.

Fading – Doppler Spread

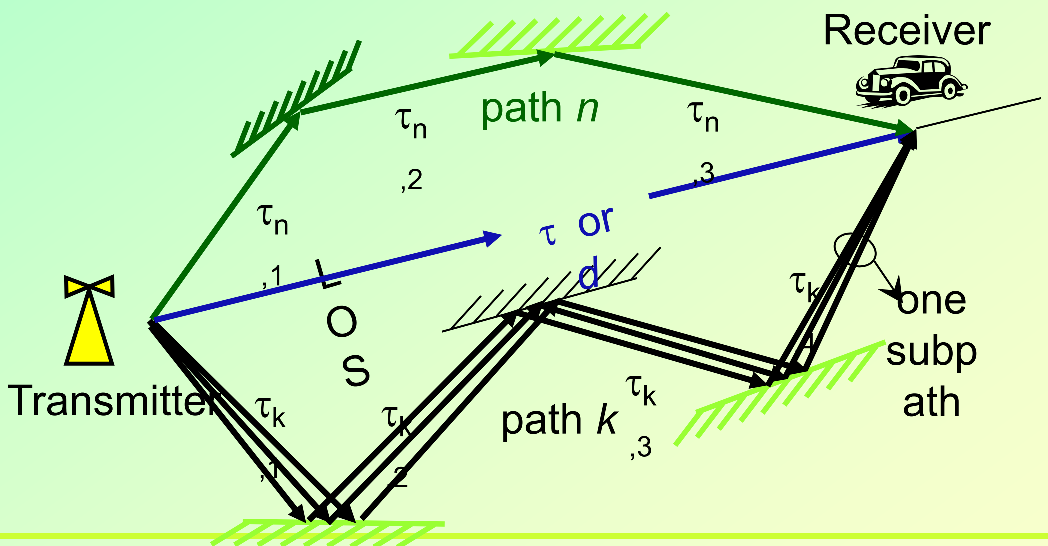
- Slow (Long) Term
- Fast (Short) Term



Exact representation of fading characteristics is not possible, because of infinite number of situation.

Fading - Slow (Long) Term

- Slower variation in the mean signal strength (distance 1-2 km)
- Channel impulse response changes slowly over a few symbol duration
- Due to:
  - Mobility
  - Terrain configuration (hill, flat area etc.)
  - The built environment (rural and urban areas etc.).
- Could be the same as flat fading if the movement of MU is very slow



Number of path

$$S_r(t) = \sum_{i=1}^{N} a_i P_0(t - \tau_i)$$

Attenuation factor for the *i*th path

Fading - Slow (Long) Term

- For N-path, the channel response is:

Phase shift

Ref. path

$$h = \sum_{i-1}^{N} \sqrt{a_i}\, e^{-j2\pi \frac{(d_i - d)}{\lambda}}$$

Wavelength

Channel gain

For large N:

- Use a statistical model, where we will have
  - random channel gains
  - random phase shifts

  Using the **Central Limit Thorem**, which states that for a very large set of random data the distribution is **Gaussian with mean and standard deviation** i.e., zero mean and σ

Fading- Fast (Short) Term

- **Describes** the constant amplitude fluctuations in the received signal as the **mobile moves** around.
- Rate of change of the channel features is larger than the rate of change of the transmitted signal (i.e., low data rates)
- Caused by *multipath reflection of transmitted signal by local scatters (houses, building etc.)*

  - random fluctuations in the received power

  - Observed over distances = $\lambda/2$
- Signal variation up to 30 dB.
- Is a frequency selective phenomenon.
- Can be described using

  - Rayleigh statistics, (*no line of sight).*

  - Rician statistics, (*line of sight).*

Fading- Fast (Short) Term - *contd.*

A received signal amplitude is given as the sum of delayed components. In terms of phasor notation it is given as:

$$S_r(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_c + \phi_i)$$

Or

$$S_r(t) = \cos(2\pi f_c t) \sum_{i=1}^{N} a_i \cos(\phi_i) - \sin(2\pi f_c t) \sum_{i=1}^{N} a_i \sin(\phi_i)$$

In-phase

Quadrature

The phase ϕ_i can be assumed to be uniformly distributed in the range (0, 2π), provided the locations of buildings etc. are completely random.

Fading- Fast (Short) Term - *contd.*

For a large *N*, the amplitude of the received signal is:

$$S_r(t) = X\cos(2\pi f_c t) - Y\sin(2\pi f_c t)$$

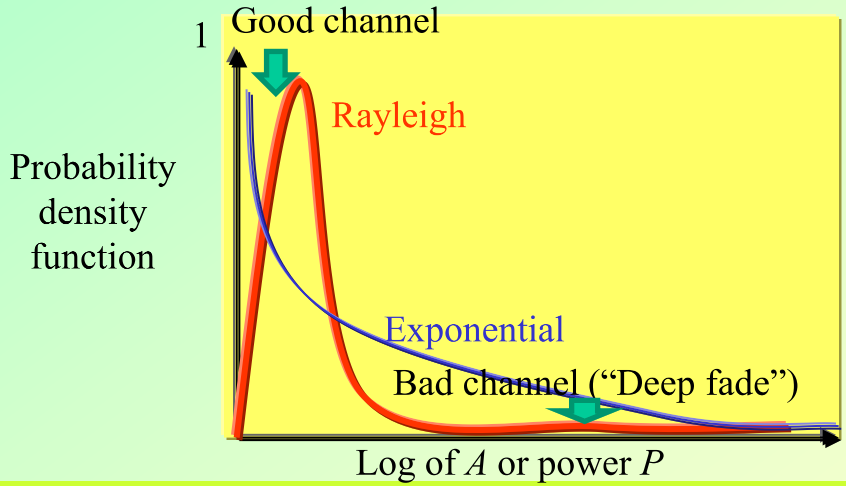
where

$$X = \sum_{i=1}^{N} a_i \cos(\phi_i), \ \ Y = \sum_{i=1}^{N} a_i \sin(\phi_i)$$

X and *Y* are independent, identically distributed Gaussian random variables.

The envelope of the received signal is:

$$A = (X^2 + Y^2)^{0.5}$$



Which is Rayleigh distributed: Assuming all components received have approximately the same power and that all are resulting from scattering.

$$p(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Where $0 < r < \infty$, σ^2 is the variance of *A* (the total received power in the multipath signal).

Mean square: $\overline{r^2} = 2\sigma^2$

Fading- Fast (Short) Term - *contd.*

Problem: For a signal with Rayleigh-distributed amplitude, what is the probability that the received signal power is at least 20 dB below the mean power

Solution: From Rayleigh distribution $$p(r)=\frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

we have $\overline{r^2} = 2\sigma^2$

A power level of 20 dB below the mean power corresponds to:

Note antilog of 20 dB = 100

$$\frac{r_{min}^2}{2\sigma^2} = \frac{1}{100}$$

The probability $\Pr(r < r_{min}) = 1 - exp\left(-\frac{1}{100}\right)$

$= 9.95\times10^{-3}$.

Fading- Fast (Short) Term - *contd.*

- The probability that the realization of the random variable has a value smaller than *x* is defined by the cumulative distribution function:

$$\mathrm{cdf}(r) = \int pdf(u)du$$

- Applying it to the Rayleigh distribution

$$\mathbf{cdf}(r) = 1 - \mathbf{exp} - (r^2 / 2\sigma^2)$$

- For small *r*

$$\mathbf{cdf}(r) \sim r^2 / 2\sigma^2$$

Ricean Fading

- If there is one direct component in addition to scattered components, the envelope of the received multipath signal is Ricean, where the impulse response has a non zero mean.

Ricean distribution = Rayleigh signal + direct line of sight signal

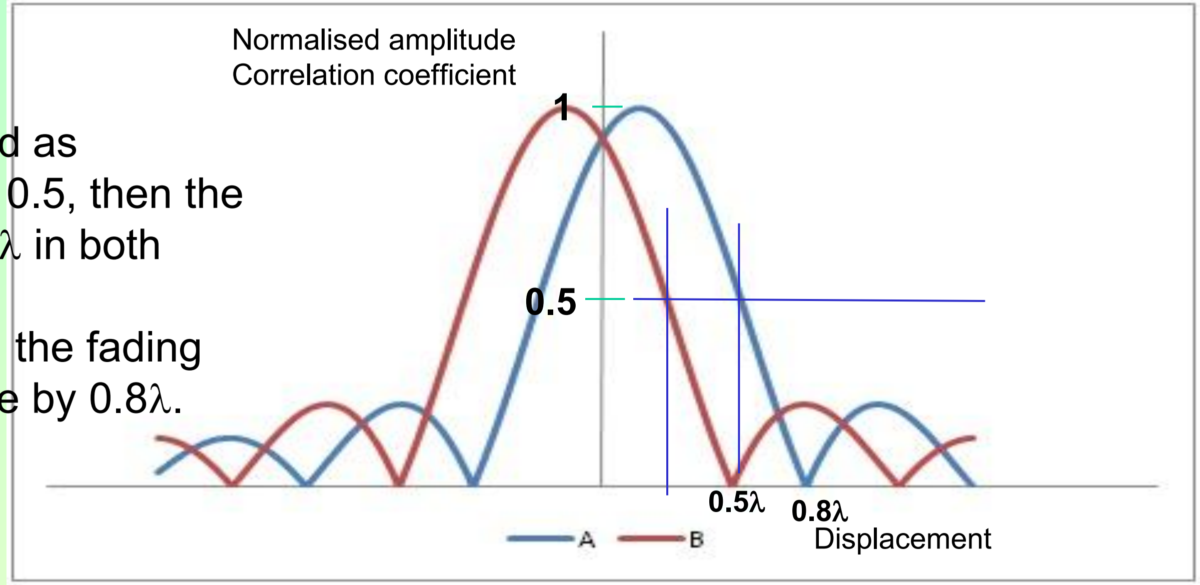
$$p(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+s^2}{2\sigma^2}\right)I_0\left(\frac{rs}{\sigma^2}\right)$$

σ^2 is the power of the line of sight signal and I_0 is a Bessel function of the first kind

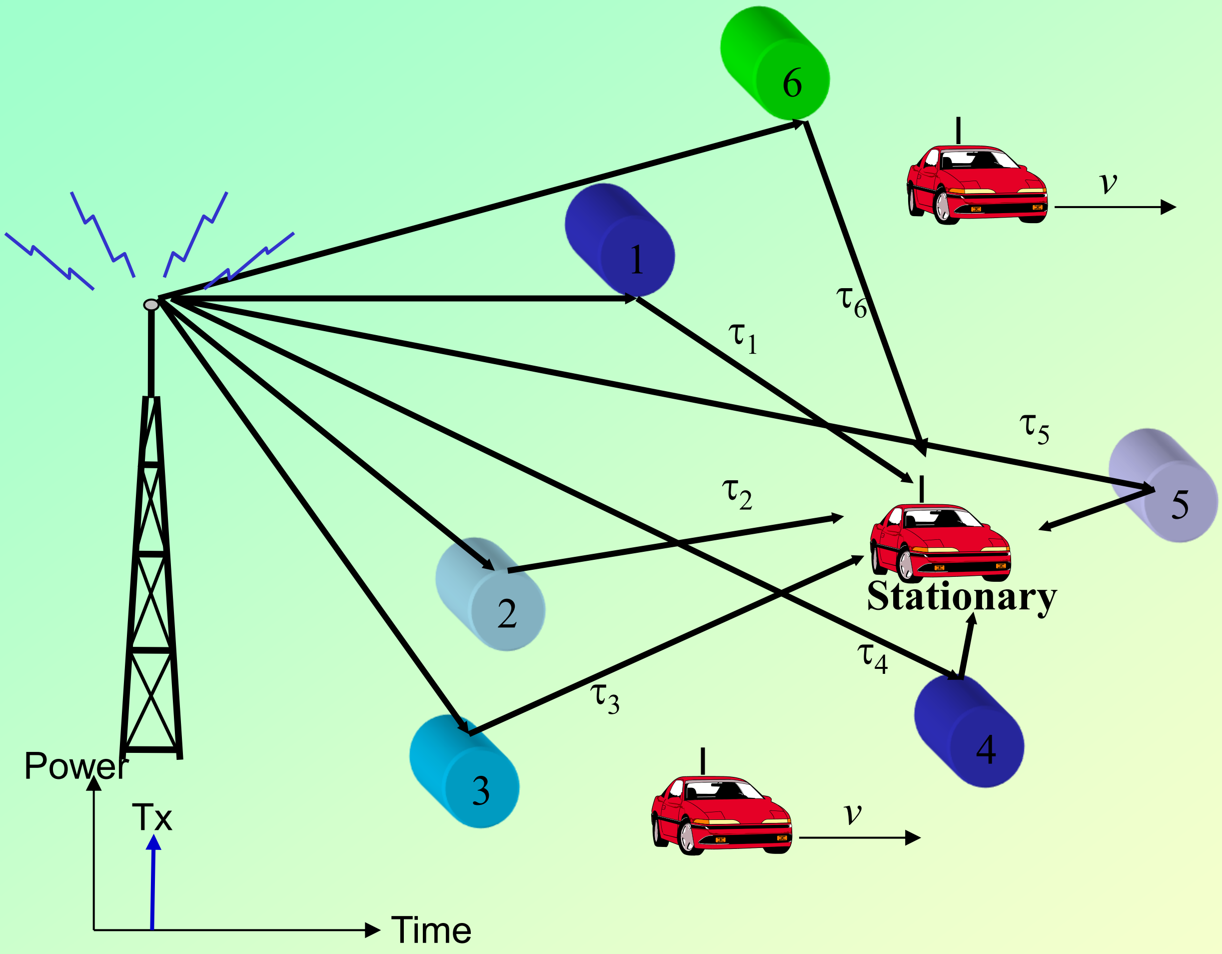
Fading - Problem

In mobile wireless communication, assume that a mobile unit is located at a fading dip. On average, what minimum distance should the mobile unit move so that it is no longer influenced by this fading dips as shown in the figures.

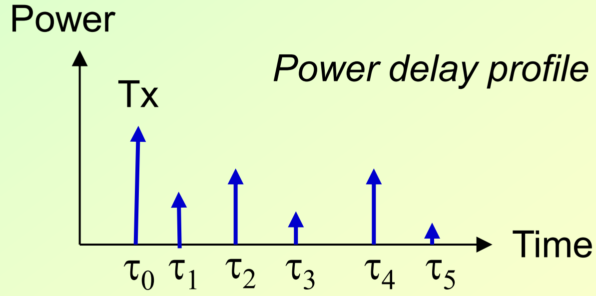
1- Considering no longer influenced as amplitude correlation coefficient of 0.5, then the mobile unit needs to move by 0.25λ in both cases.
2- For complete decorrelation from the fading dips, the the mobile unit must move by 0.8λ.

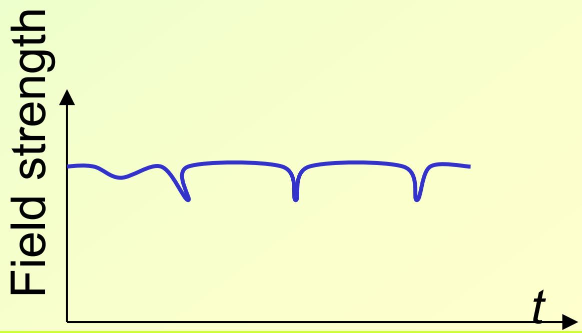


Fast Fading – Cases 1: Stationary Mobile



The maximum delay for which the received power level becomes negligible is known as **the maximum delay spread** τ_{max}. Sometimes Root-Mean-Square value τ_{rms} is used.





Fast Fading – Cases 1

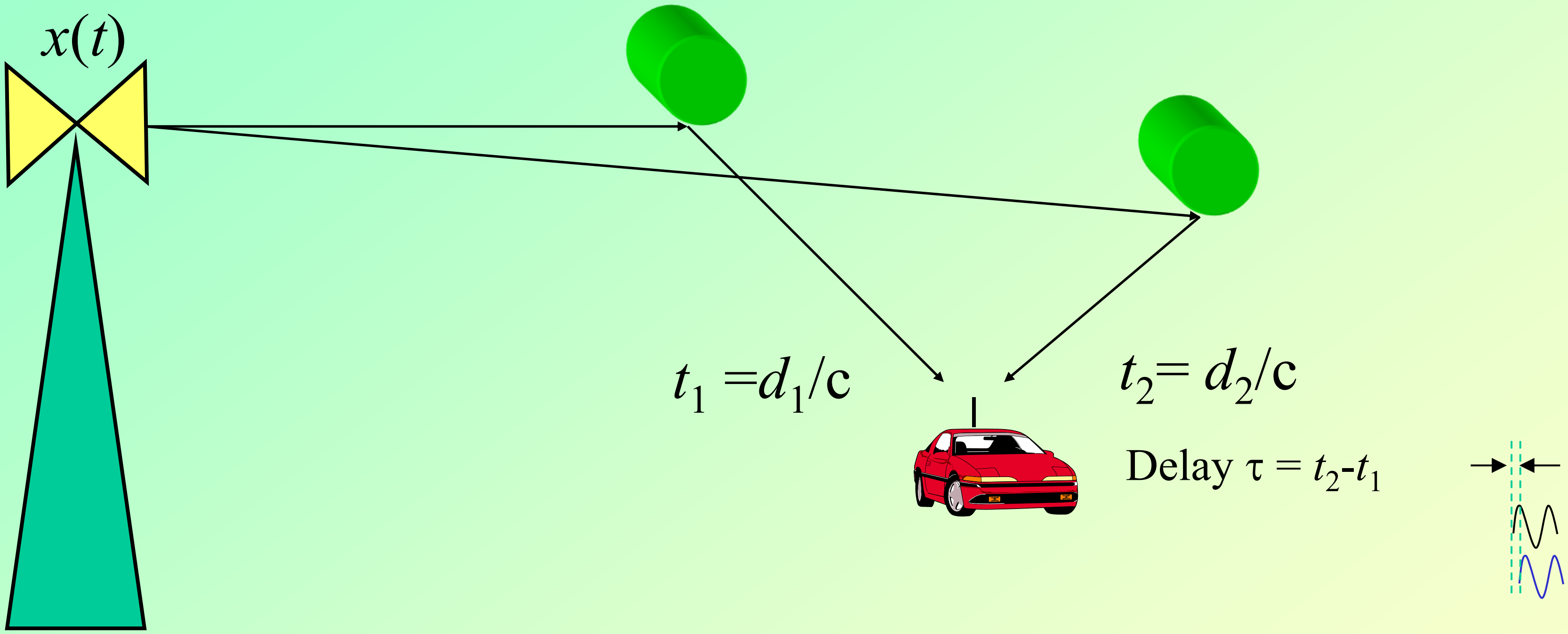
- The number of fading depends on:
  - Traffic flow
  - Distance between the BS and the moving MU
- The received signal at the MU is:

$$S_r(t) = h(t,\tau) \otimes p(t) + n(t)$$
$$= h(t)p(t) + n(t)$$

$$S_r(t) = \sum_{i=1}^{N} a_i P_0(t - \tau_i)$$

Assuming no noise

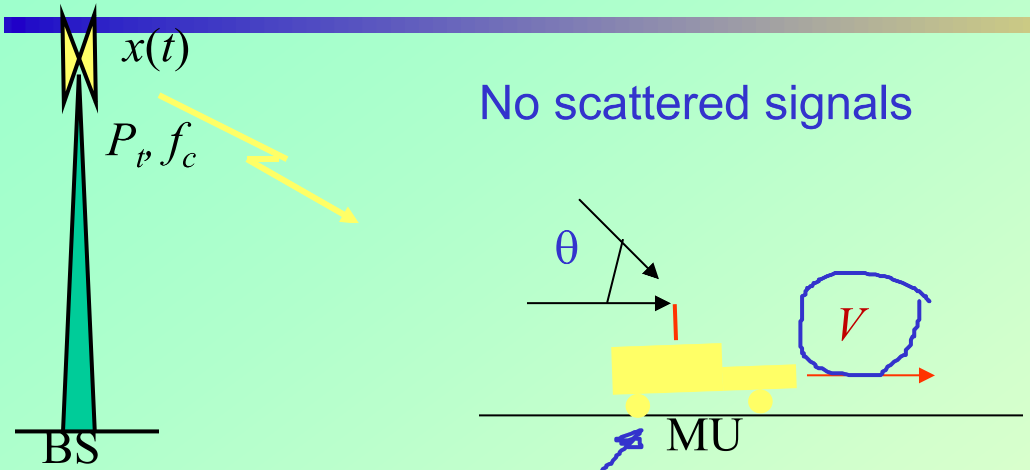
Fast Fading – Cases 2

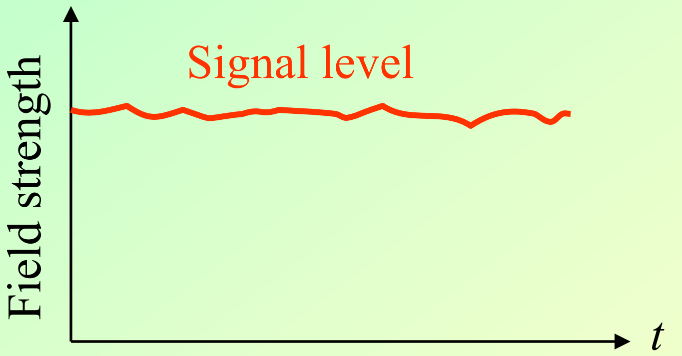


$$S(t) = x(t)\exp(j\phi_c)\exp(j\omega_c t)$$

$$x(t) = \sum_{i=1}^{N} a_c a_i(t)\exp[-j\omega_c \tau_i(t)]$$

Fast Fading – Cases 3: Non-stationary Mobile





The received signal at the mobile is:

$$s_r(t) = a_o e^{j(2\pi f_c + \phi_o - \beta x \cos\theta)}$$

$x = Vt$

Amplitude

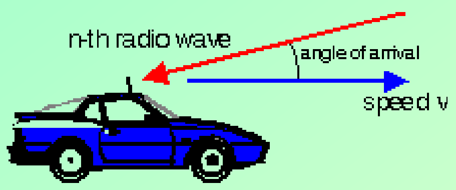
Wave number $= 2\pi/\lambda$

Transmitting carrier frequency

Fast Fading – Cases 3: Doppler Frequency

A moving object causes the frequency of a received wave to change

Substituting for β and *x*, the expression for the received signal is



$$s_r(t) = a_o e^{j2\pi\left(f_c - \frac{V}{\lambda}\cos\theta\right)t}$$

The Doppler frequency

$$f_D = \frac{V}{\lambda}\cos\theta = f_m \cos\theta$$

The received signal frequency

$$f_r = f_c - f_m \cos\theta$$

Fast Fading – Cases 3: Doppler Frequency

- When $\theta = 0^o$ (mobile moving away from the transmitter)

$$f_r = f_c - f_m$$

- When $\theta = 90^o$ (I.e. mobile circling around)

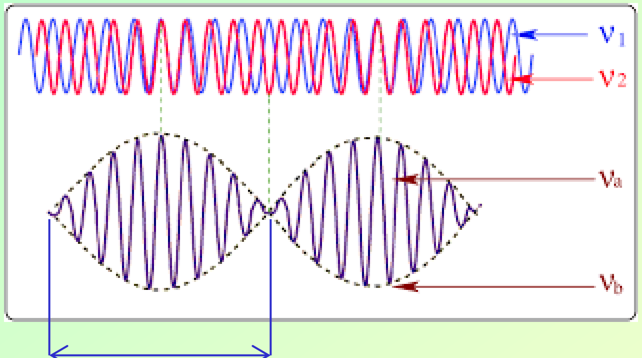
$$f_r = f_c$$

- When $\theta = 180^o$ (mobile moving towards the transmitter)

$$f_r = f_c + f_m$$

Effects of Doppler shifts

- Bandwidth of the signal could increase or decrease leading to poor and/or missed reception.
- If all waves are Doppler shifted by the same amount, i.e., 100 Hz, the effect on the system would be negligible. The receiver can deal with the shift.
- If waves experience different Doppler shifts, then superposition of Doppler shifted waves creates maxima and minima (dips).



$$\frac{1}{|f_{D1} - f_{D2}|}$$

The beat frequency = Is the difference between two Doppler frequencies

- So superposition of many Doppler shifted waves leads to the phase shifts → random frequency modulation of the received signal.

Effects of Doppler shifts

- The effect in time is coherence time variation and signal distortion
  - ***Coherence time:*** **Time duration over which channel impulse response is invariant, and in which two signals have strong potential for amplitude correlation, and is expressed by:**

$$T_{co} = \sqrt{\frac{9}{16\pi f_{D\text{-}max}^{2}}}$$

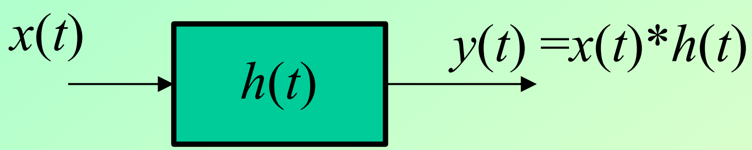
  - **where $f_{D\text{-}max}$ is the maximum Doppler shift, which occurs when $\theta = 0$ degrees**

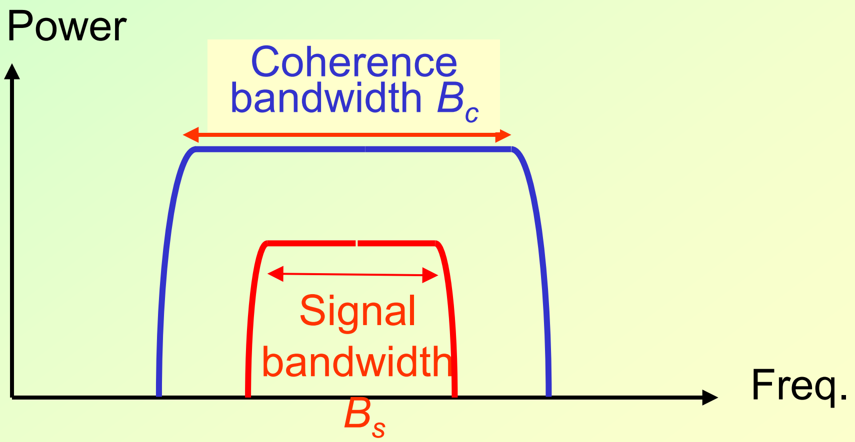
- To avoid distortion due to motion in the channel, the symbol rate must be greater than the inverse of coherence time.

Coherence Bandwidth

Coherence Bandwidth

- Range of frequency over which channel is "flat"
- Is a statistical measure of the range of frequencies over which the channel passes all spectral components with approximately equal gain and linear phase.



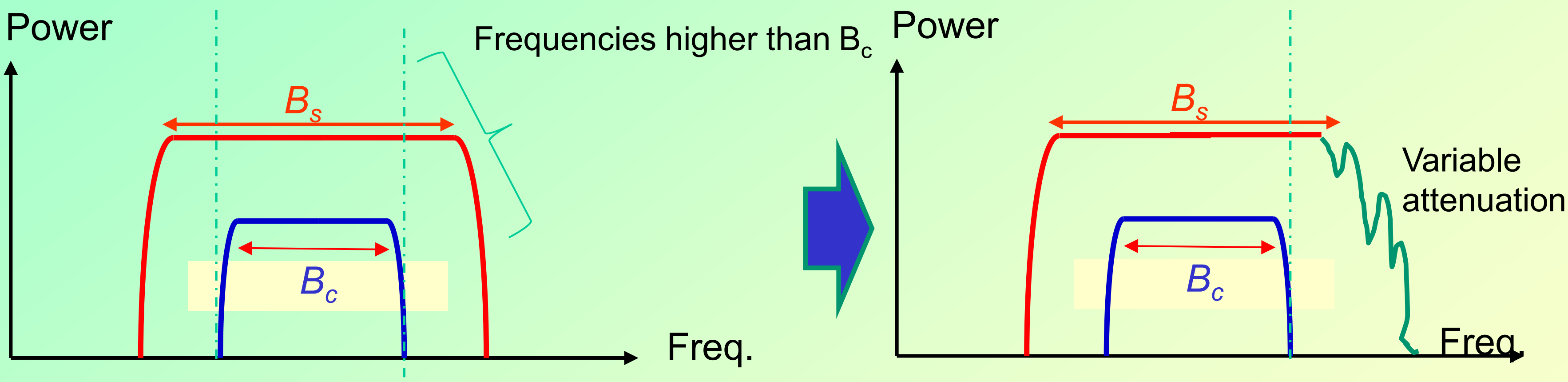


- If signal bandwidth B_s<< the channel or coherence bandwidth B_c, then there is no distortion, and is known as **Flat Fading** that results in a **burst of error**.

\- **Error correction coding** is used to overcome this problem.

Frequency Selective Fading

- If B_s>> B_c, then there is distortion, i.e., **inter-symbol interference (ISI)** in time domain and **frequency selective fading** in the frequency domain



Thus, Variable attenuation outside the BC band leads to signal distortion →Frequency selective fading

Note:, B_s= 1/T_s > B_c,= 1/τ_t, where τ_t is the time between the 1st and last received signal components i.e., the **maximum excess delay**.

Therefore, $\tau_t > T_s$ → ISI

Estimation of Coherence Bandwidth

For correlation > 0.9 $B_c \approx \frac{0.2}{\tau_t}$

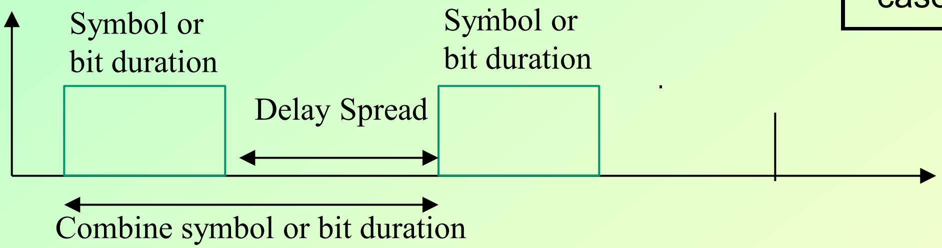
For correlation > 0.5 $B_c \approx \frac{0.02}{\tau_t}$

For picocells 1 .. 2 GHz:

- *Delay spread* < 50 - 300 nsec

- Home 50 nsec
- Shopping mall 100 - 200 nsec
- Railway station 200 - 450 nsec
- Office block 100 - 400 nsec

Delay spread figures at 900 MHz	Delay in microseconds
Urban	1.3
Urban, worst-case	10 - 25
Suburban, typical	0.2 - 0.31
Suburban, extreme	1.96 - 2.11
Indoor, maximum	0.27
Delay Spread at 1900 MHz	
Buildings, average	0.07 - 0.094
Buildings, worst - case	1.47



Thus, the delay spread → reduced channel efficiency

Channel Classification

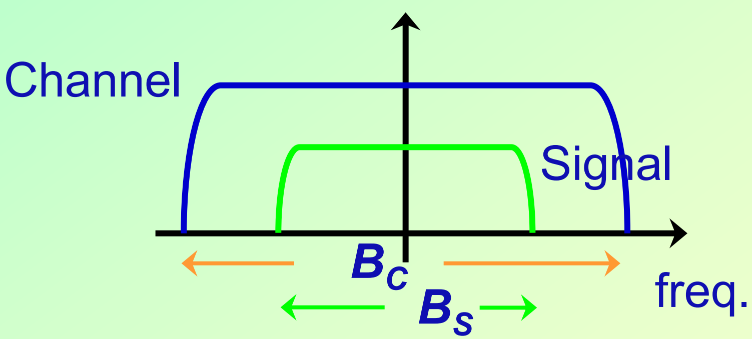
Based on Time-Spreading

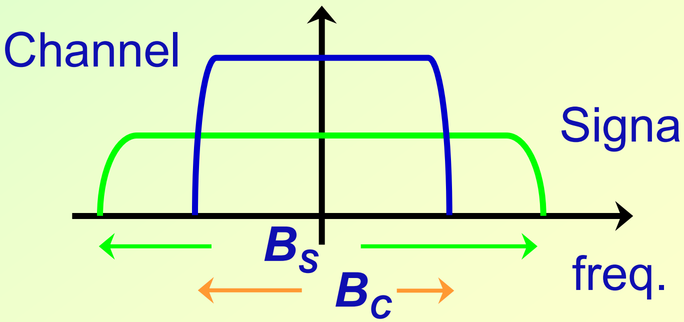
Flat Fading

1. $B_S < B_C \Leftrightarrow T_m < T_s$
2. Rayleigh, Ricean distrib.
3. Spectral chara. of transmitted signal preserved

Frequency Selective

1. $B_S > B_C \Leftrightarrow T_m > T_s$
2. Intersymbol interference
3. Spectral chara. of transmitted signal not preserved
4. Multipath components resolved



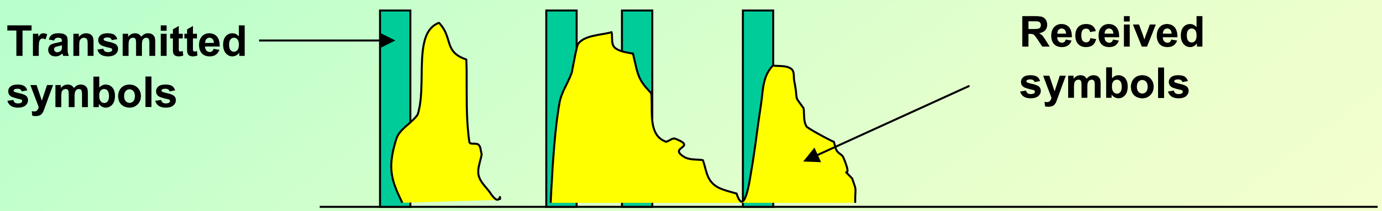


System Performance - BER

- In conventional digital communications with no fading, the BER decreases approximately linearly with the increasing SNR.
- In fading channels:
  - the SNR is not constant
  - The probability of the link in a Deep Fading (i.e., where the SNR is low) dominants the BER performance
  - Thus, the average BER decreases only linearly with increasing SNR.
  - Therefore, improving the BER is not achieved by simply increasing the **transmit power**.

System Performance - Dispersion

- The delay spread limits the maximum data rate:
  - No new impulses should arrive at the receiver before the last replica of the pervious impulse has perished.
  - Otherwise, symbol spreads (dispersion) into its adjacent slot, thus resulting in **intersymbol interference (ISI) → more errors,** which cannot be reduced by simply increasing the transmit power



- ISI is essentially determined by the ratio between the symbol duration and the duration of impulse response of the channel
- The signal arrived at the receiver directly and phase shifted
  - Distorted signal depending on the phases of the different parts

System Design and Performance Prediction

- Base station placement dependent on
  - Propagation environment
  - Anticipated geographic distribution of users
  - Economic considerations (minimize number of base stations)
  - Political and public opinion considerations
  - Traffic types (3G)
- Performance figure of merit
  - Spectrum efficiency for voice: η_v voice circuits/MHz/base station
  - Spectrum efficiency for information: η_i bps/MHz/base station
  - Dropped call rate – fraction of calls ended prematurely

Summary

- The random fluctuations in the received power are due to fading.
- If there is a relative motion between transmitter and receiver (mobile) the result is Doppler shift
- If maximum Doppler shift is less than the data rate, there is "slow" fading channel.
- If maximum Doppler shift is larger than the data rate, there is "fast" fading channel.

Next Lecture: Diversity