Optical Fibre Communication Systems

Lecture 2: Nature of Light and Light Propagation

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Wave Nature of Light

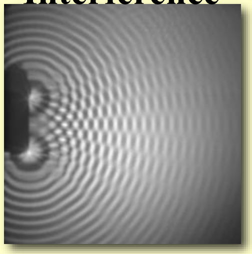
• **Newton (1680) b**elieved in the **particle theory** of light. In **reflection** and **refraction**, light behaved as a particle. He explained the straight-line casting of sharp shadows of objects placed in a light beam. But he could not explain the textures of shadows

• **Young (1800)** – Showed that light interfered with itself. **Wave theory:** Explains interference, diffraction, and dispersion.

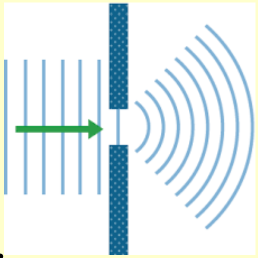
• **The wave theory** is also able to account for the fact that the edges of a shadow are not quite sharp.

This theory describes: *Propagation, reflection, refraction* and *attenuation*

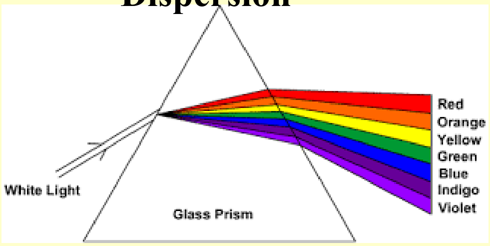
Interference

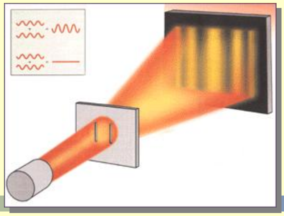


Diffraction



Dispersion





G Ekspong, Stockholm University, Sweden, 1999.

Wave Nature of Light - *contd.*

James Clerk Maxwell (1850) - Mathematical theory of electromagnetism led to the view that **light is of electromagnetic nature, propagating as a wave** from the source to the receiver.

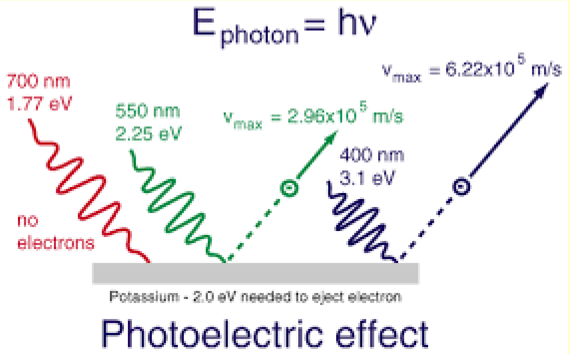
Heinrich Hertz (1887) - Discovered **experimentally the existence of electromagnetic waves at radio-frequencies**.

Wave theory does not describe the absorption of light by a photosensitive materials

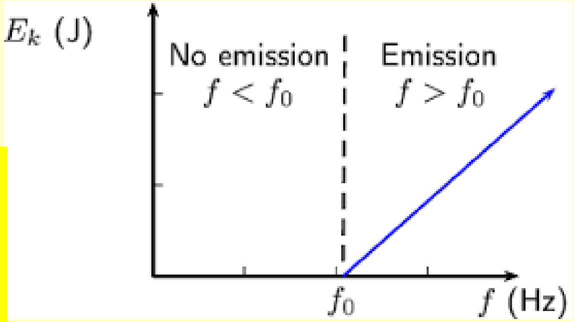
He discovered the photoelectric effect. So what is it?

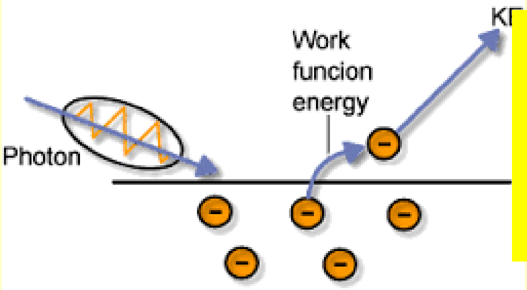
Wave Nature of Light - *contd.*

Light (electromagnetic wave) shining on a metal surface results in its **energy** being transferred to the **electrons** on the surface of the metal and some **electron will escape**. The released electron are called "**photo-electron**".



$E_{\text{photo-electron}}$ is related to f_{incident} light but **not its intensity**. So light is energetic enough to shift electron(s) or not.

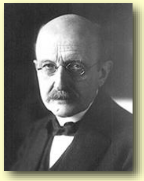




The work function – The amount of E needed by the EM wave to free an electron from a meta surface (i.e., a $E_{\text{threshold}}$).

1900-20 Max Planck, Neils Bohr and Albert Einstein
Invoked the idea of light being emitted in tiny pulses of energy.

Wave Nature of Light - *contd.*

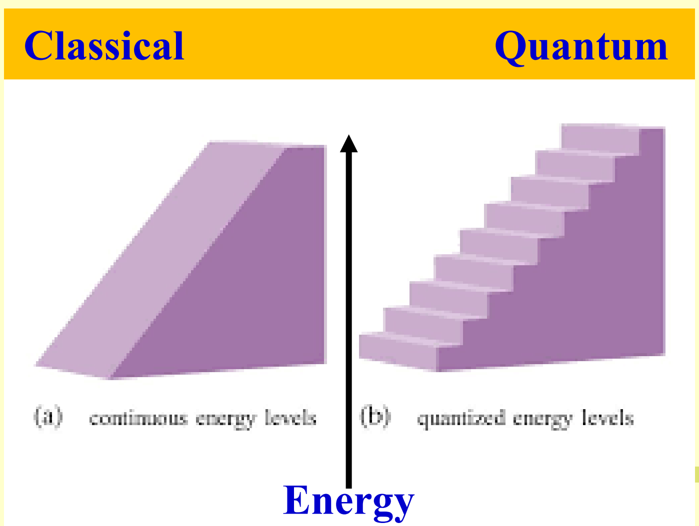


Planck (1900) - Developed a model that explained light as a **quantization of energy [Got a Nobel Prize]**.

Considered the Block Body cavity:

- E = nhf *n = 0, 1, 2, 3.., h = Plancks' constant 6.626 x* 10^{-34} *JS [or 4.136 x* 10^{-15} *eVs] , f = frequency of one of the standing wave within a black body.*

- The energy spectrum of the standing wave within the black body cavity is not continuous as in the classical theory, but take specific values:



E.g., - A standing wave within a black body cavity has a frequency *f* = 7.25 x 10^{14} Hz (Blue-violet).
The energy is: $E = n(4.136 \times 10^{-15}$ eVs$)(7.25 \times 10^{14}$ Hz$) = n(3$ eV),

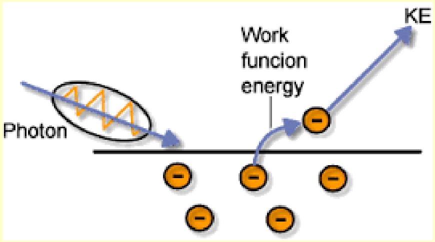
So for *n* = 1, 2, 3, 4, .. we have *E* = 3, 6, 9, 12, …. But not for 1, 2, 4, 5, 7, 8 and so on.

Interesting!

This contradicted the continuous wave theory of light, where wave DO NOT have distinct constituent!

Wave Nature of Light - *contd.*

Einstein (1905) – Used Plank's idea to showed that, in the photoelectric effect (light causing electrons to be emitted from a metal surface) **light must act as a particle**.



He came up with this:

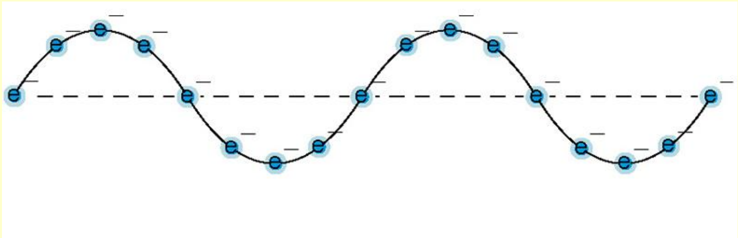
$$K_{max} = hf - \phi$$

Max. kenetic Energy of an electron

Work function

Therefore, light must be regarded as having a dual nature;

- **in some cases light acts as a wave**
- **in others it acts like a particle.**



Particle Nature of Light

Light behaviour can be explained in terms of the amount of energy imparted in an interaction with some other medium. In this case, a beam of light is composed of a stream of small lumps or QUANTA of energy, known as PHOTONS. Each photon carries with it a precisely defined amount of energy defined as:

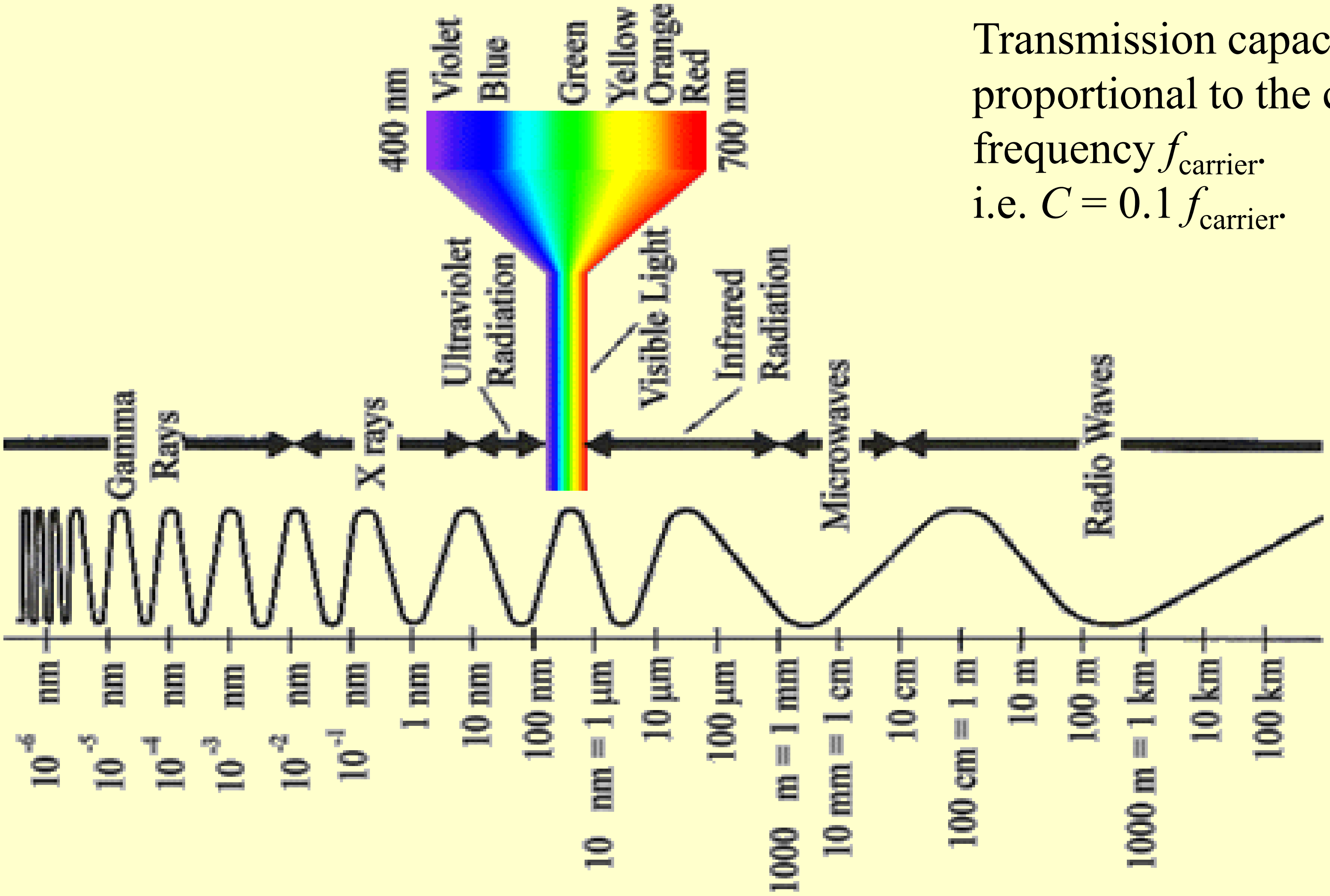
$$W_p = h*f \qquad \textbf{Joules (J)}$$

where; h = Plank's constant = 6.626×10^{-34} J.s, f = Frequency Hz

The convenient unit of energy is electron volt (eV), which is the kinetic energy acquired by an electron when accelerated to 1 eV = 1.6×10^{-19} J.

- Even though a photon can be thought of as a **particle of energy** it still has a fundamental wavelength, which is equivalent to that of the propagating wave as described by the wave model.
- **This model of light is useful when the light source contains only a few photons.**

Electromagnetic Spectrum



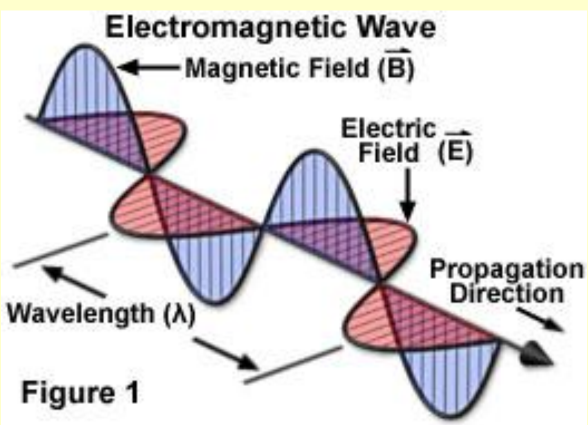
Transmission capacity C is proportional to the carrier frequency $f_{carrier}$.
i.e. $C = 0.1 f_{carrier}$.

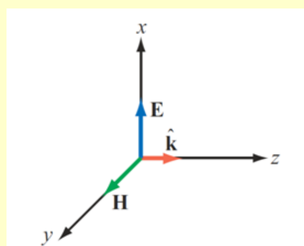
Electromagnetic Radiation

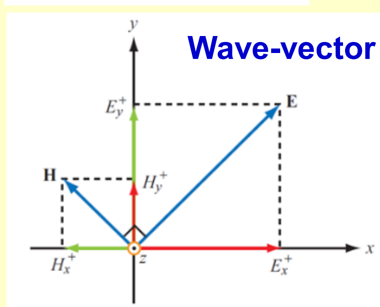
- Carries energy through space (includes visible light, dental x-rays, radio waves, heat radiation from a fireplace)
- The wave is composed of a combination of mutually perpendicular electric and magnetic fields the direction of propagation of the wave is at right angles to both field directions, this is known as an:

ELECTROMAGNETIC (EM) WAVE

EM wave move through a vacuum at 3.0×10^8 m/s ("speed of light")







$$\boldsymbol{E} = \boldsymbol{E}_0(z,\phi)e^{j(\omega t - \beta z)}$$

$$\boldsymbol{H} = \boldsymbol{H}_0(z,\phi)e^{j(\omega t - \beta z)}$$

$$E = v_p H = [c/n]H$$

Speed of light in a vacuum $c = f \times \lambda_o$

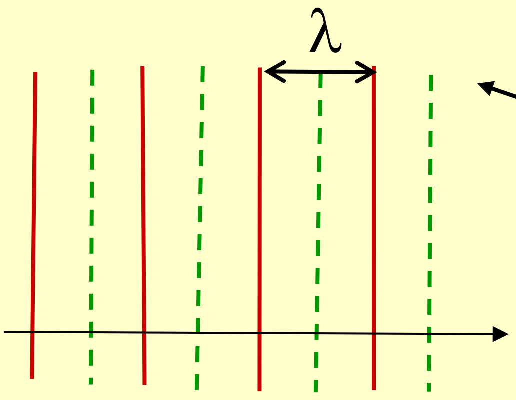
β - Propagation constant = ω/v_p

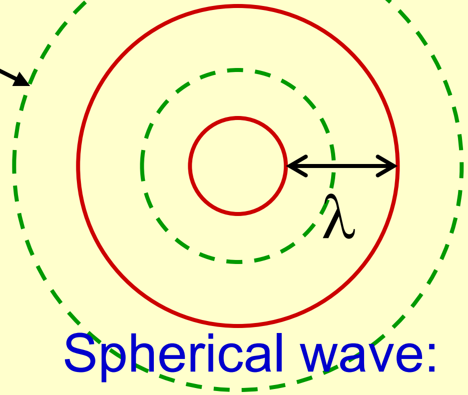
The Wave Equation

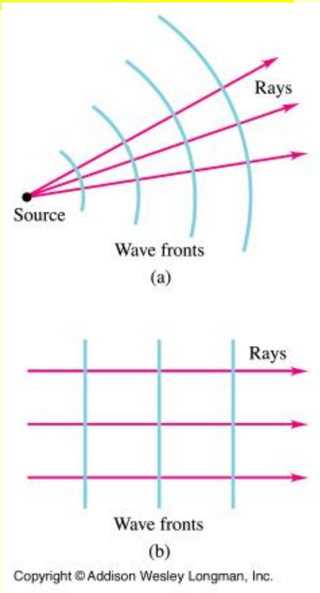
Solutions to Maxwell's equations:

Ideal isotropic radiation

phase fronts







Spherical wave:

Plane wave: At a large distance from a source.

Constant phase → Wave-front is a plane

$$y(z,t) = A\sin(kz - \omega t - \emptyset)$$

Wave number in vacuum

$$k = \frac{2\pi}{\lambda}$$

$$k = n \cdot k_0$$

$$\lambda = \lambda_0 / n$$

Note: $k = \beta$

In complex form

$$y(z,t) = A\,e^{j(kz-\omega t-\emptyset)}$$

$$k = \omega\sqrt{\varepsilon\mu_0} = \sqrt{\varepsilon_r}\cdot\omega\sqrt{\varepsilon_0\mu_0} = \sqrt{\varepsilon_r}\cdot k_0$$

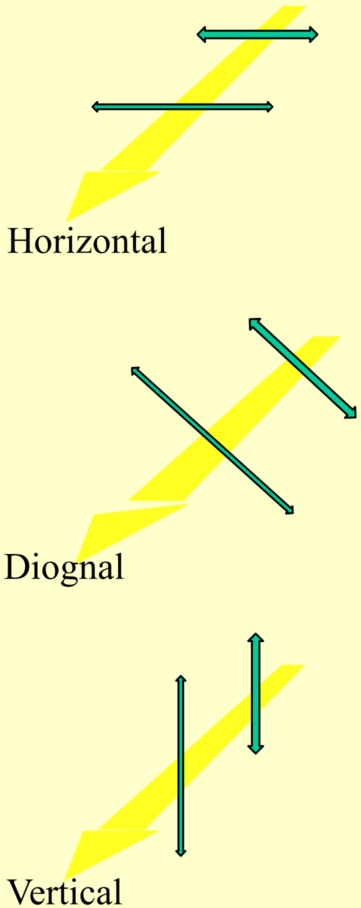
$$n = \sqrt{\varepsilon_r}$$

$A\,e^{-j\emptyset}$ Complex amplitude or "phasor"

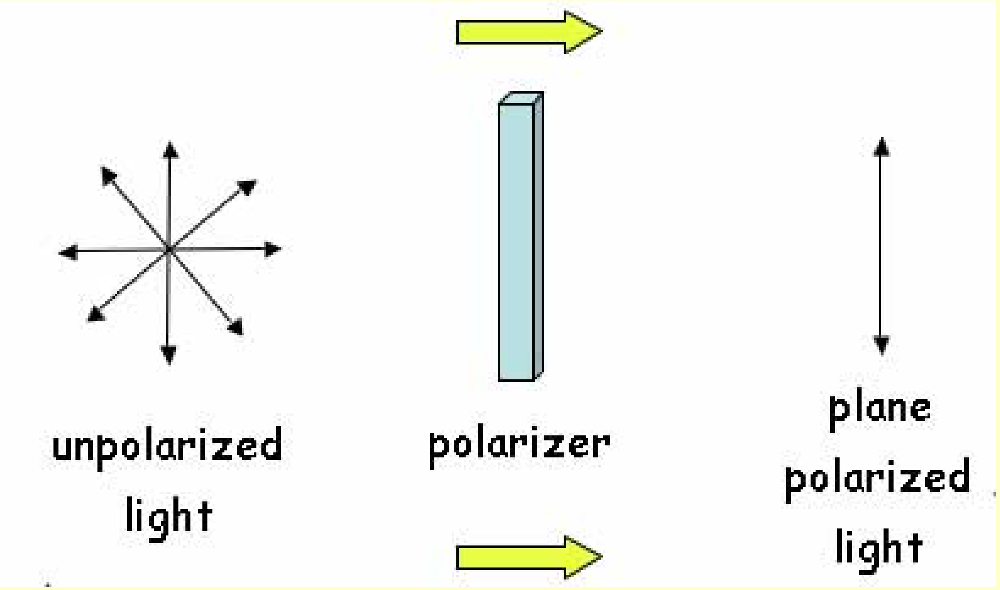
Polarization

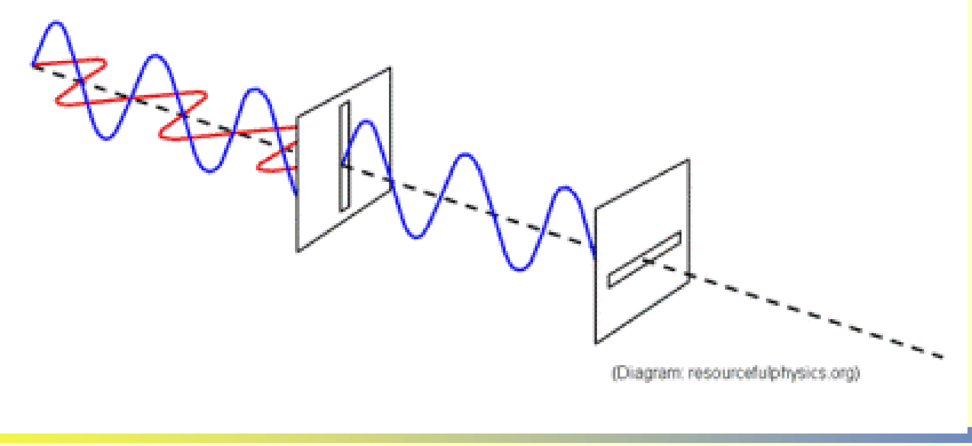
- Demonstrates the transverse nature of EM wave.
- Un-polarized light source: The electric field is vibrating in many directions; all perpendicular to the direction of propagation.
- Polarized light source: The vibration of the electric field is mostly in one direction. Any direction is possible as long as it's perpendicular to the propagation.

- Types of polarization - Depending on existence and changes of different electric fields
  - Linear
    - Horizontal (*E* field changing in parallel with respect to earth's surface)
    - Vertical (*E* field going up/down with respect to earth's surface)
    - Dual polarized
  - Circular (E_x and E_y)
    - Similar to satellite communications
    - Tx and Rx must agree on direction of rotation
  - Elliptical
  - Linear polarization is used in WiFi communications



Polarization



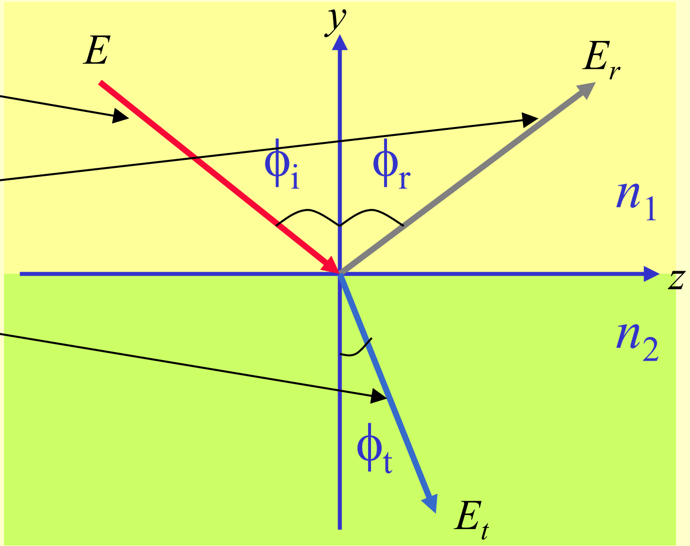


One Dimensional EM Wave

Incident wave $\boldsymbol{E} = \boldsymbol{E}_0(z,\phi)e^{j(\omega t - \beta z)}$

Reflected wave $\boldsymbol{E_r} = \boldsymbol{E}_{0r}(z,\phi)e^{j(\omega_r t - \beta_r z)}$

Transmitted wave $\boldsymbol{E_t} = \boldsymbol{E}_{0t}(z,\phi)e^{j(\omega_t t - \beta_t z)}$



$n_1 < n_2$

Note, phases of the three waves, which are independent of *z*, *t*, must be equal, i.e.,:

$$(\omega \; t - \beta z) = (\omega_r t - \beta_r z) = (\omega_t t - \beta_t z)$$

At the boundary point i.e., $z = 0$, we have: $\omega \; t = \omega_r t = \omega_t t$

Or $\omega = \omega_r = \omega_t$

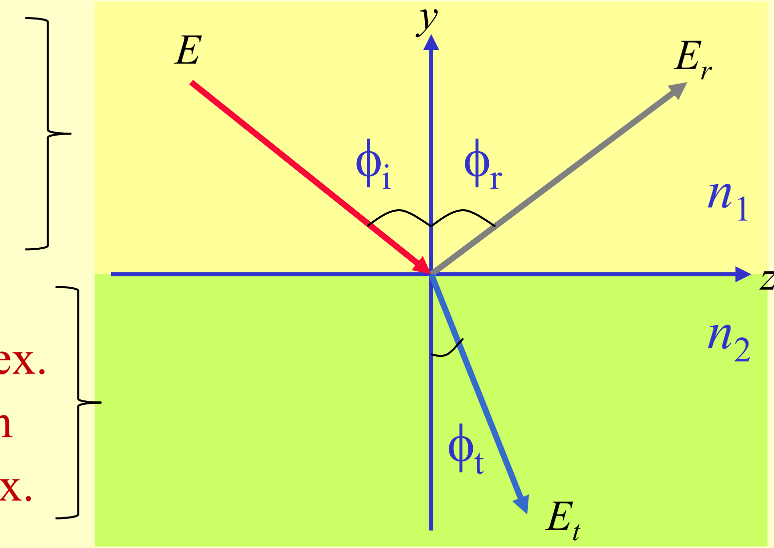
Properties of Light

Law of Reflection

- External reflection: $n_1 < n_2$
- External reflection: $n_1 > n_2$

Law of Refraction -

- Light beam is bent towards the normal when passing into a medium of higher refractive index.
- Light beam is bent away from the normal when passing into a medium of lower refractive index.



Index of Refraction

- n = Speed of light in a vacuum / Speed of light in a medium

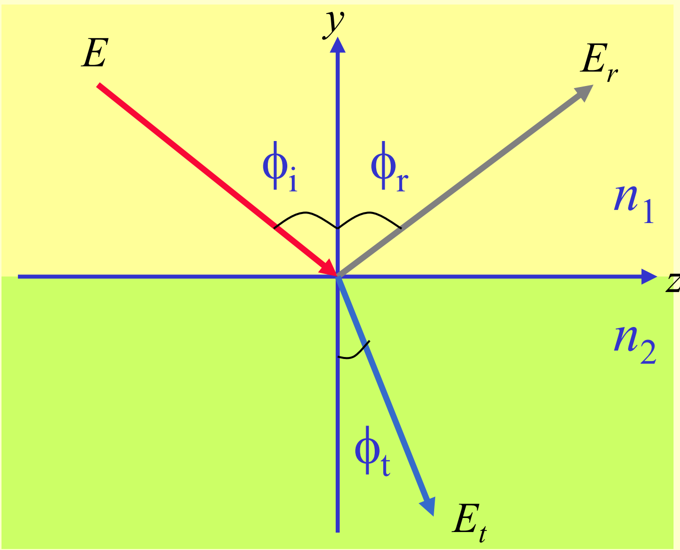
Inverse square law

- Light intensity diminishes with the square of distance (d^2) from the source.

Law of Reflection

Considering at time $t = 0$ within the boundary plane, we have

$$-\beta z = -\beta_r z = -\beta_t z$$



With the reflected and refracted waves being in the plane of incident, and if

$$\beta z = \beta_r z$$

Then we have

$$\beta z \sin\phi_i = \beta_r z \sin\phi_r$$

Note that, both waves are travelling in the same medium. Therefore have identical wavelength, so $\beta = \beta_r$.

Thus

$$\phi_i = \phi_r$$

Incident angle = Reflection angle

Law of Refraction

Similarly, we have

$$\beta \boldsymbol{z} = \beta_{\boldsymbol{t}} \boldsymbol{z}$$

Then we have

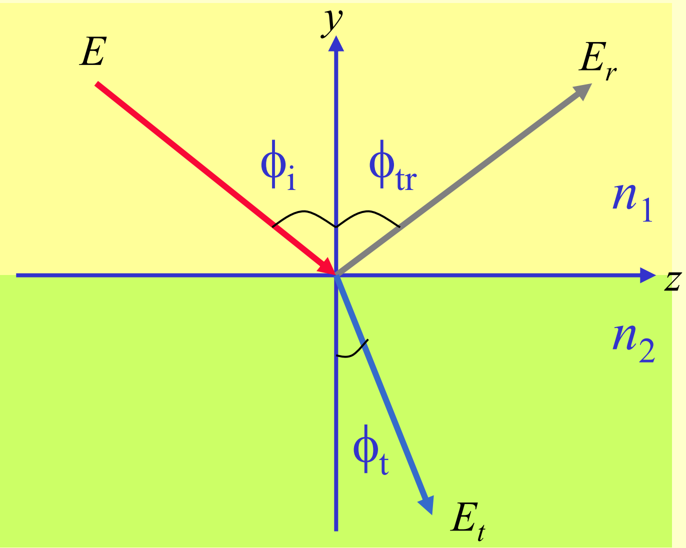
$$\beta z \sin\phi_i = \beta_t z \sin\phi_t$$

So we have

$$\beta = \frac{\omega}{v_p} = \frac{n_1}{c}\omega$$

$$\beta_{\boldsymbol{t}} = \frac{\omega}{v_{pt}} = \frac{n_2}{c}\omega$$

One Dimensional EM Wave – Boundary Conditions



It applies to the instantaneous values of the fields at the boundary.

Note that $E = v_p H = (c/n)H$

In each medium, we can write the following equations, assuming that $\phi_i = \phi_r$

TE: $$\begin{cases} E + E_r = E_t \\ n_1 E\cos\emptyset_i - n_1 E_r \cos\emptyset_i = n_2 E_t \cos\emptyset_t \end{cases}$$

TM: $$\begin{cases} n_1 E + n_1 E_r = n_2 E_t \\ E\cos\emptyset_i - E_r \cos\emptyset_i = E_t \cos\emptyset_t \end{cases}$$

Reflectance and Transmission

The reflection and transmission coefficients:

$$\rho_r = \frac{E_r}{E} \qquad \text{and} \qquad \rho_t = \frac{E_t}{E}$$

Reflectance – Is the fraction of the power *P* in the incident wave that is reflected

$$R = \frac{P_r}{P} = \left(\frac{E_r}{E}\right)^2 = \rho_r^2$$

Transmittance – Is the fraction of the power *P* in the incident wave that is transmitted

$$T = \frac{P_t}{P} = 1 - R\frac{n_2}{n_1}\left(\frac{\cos \emptyset_t}{\cos \emptyset_i}\right)\rho_t^2$$

Phase Changes

External Reflection

Note, in certain cases we have:

$$E_r = -|\rho_r|E$$

I.e., a phase change of π

$$\boldsymbol{E_r} = e^{j\pi}\boldsymbol{E}_{0r}(z,\phi)e^{j(\omega_r t - \beta_r z)}$$

$$\boldsymbol{E_r} = \boldsymbol{E}_{0r}(z,\phi)e^{j(\omega_r t - \beta_r z - \pi)}$$

TE: Phase change takes place for any angel of incidence

TM: Phase change occurs for $\phi_i > \phi_B$, where ϕ_B is the Brewster angle

Phase Changes

Internal Reflection

- For $0 < \phi_i < \phi_c$
  - TE: No phase change
  - TM: Phase change of π for $\phi_i < \phi'_B$
- For $\phi_i > \phi_c$
  - TE: The phase change

$$\Delta\emptyset = -2\tan^{-1}\left\{\frac{\sqrt{\sin^2\emptyset_i - n^2}}{\cos\emptyset_i}\right\}, \text{ where } n = n_2/n_1$$

  - TM: The phase change

$$\Delta\emptyset = -2\tan^{-1}\left\{\frac{\sqrt{\sin^2\emptyset_i - n^2}}{n^2\cos\emptyset_i}\right\}, \text{ where } n = n_2/n_1$$

Group Velocity

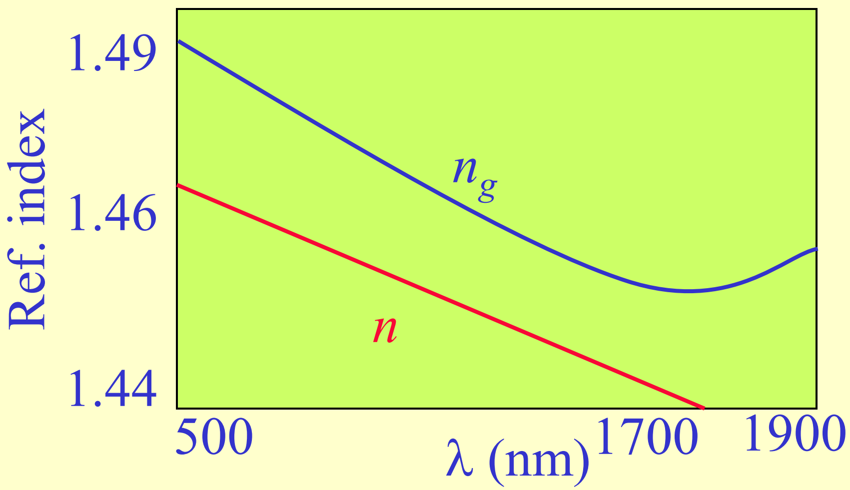
- A pure single frequency EM wave propagate through a wave guide at a

  Phase velocity $v_p = c / n$

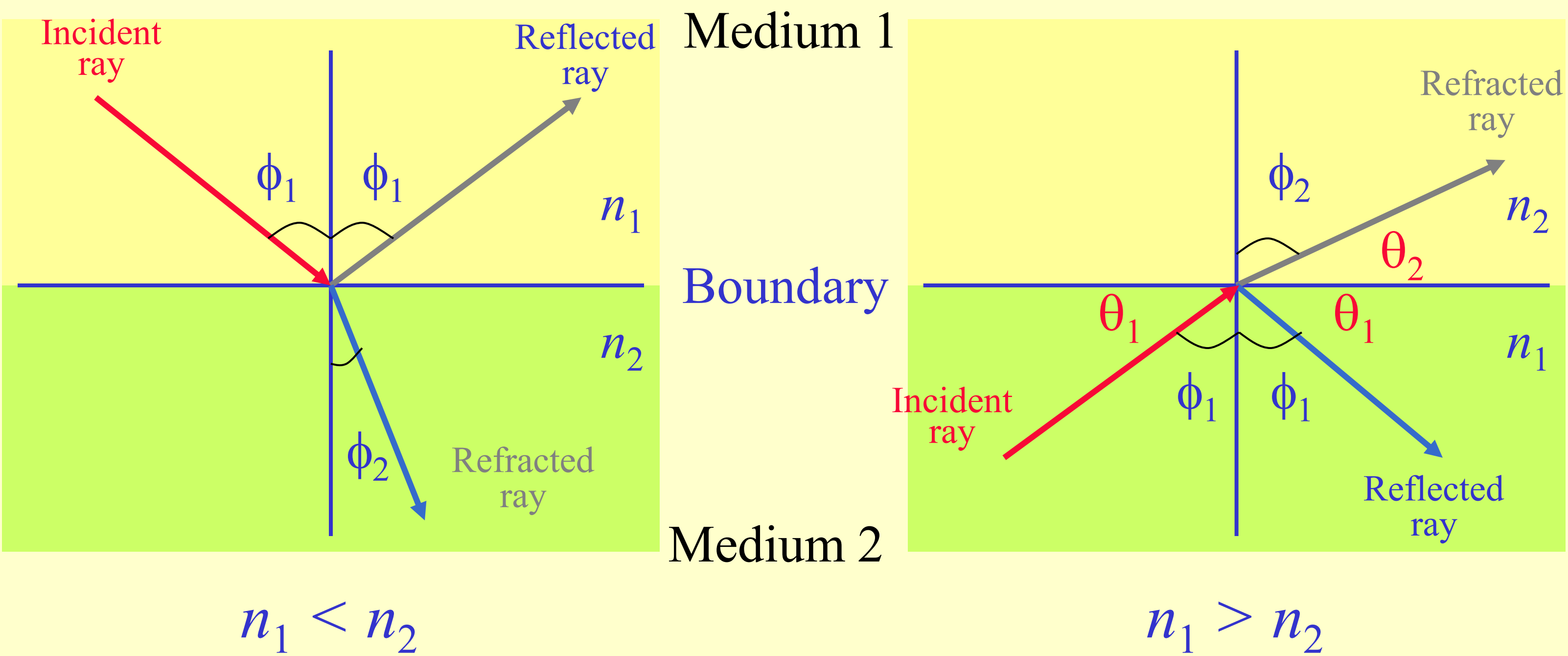
- However, non-monochromatic waves travelling together will have a velocity known as **Group Velocity** $v_g = c / n_g$

Where the fibre group index is:

$$n_g = n - \lambda \frac{dn}{d\lambda}$$



Reflection and Refraction of Light



Using the **Snell's law** at the boundary we have:

$$\boldsymbol{n_1 \sin \phi_1 = n_2 \sin \phi_2}$$

or

$$\boldsymbol{n_1 \cos \theta_1 = n_2 \cos \theta_2}$$

ϕ_1 = The angle of incident

Total Internal Reflection

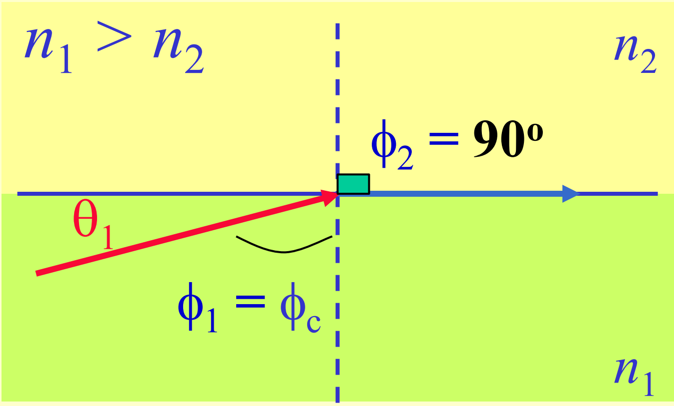
- As ϕ_1 **increases (or θ_1 decreases) the reflected ray approaches the boundary**

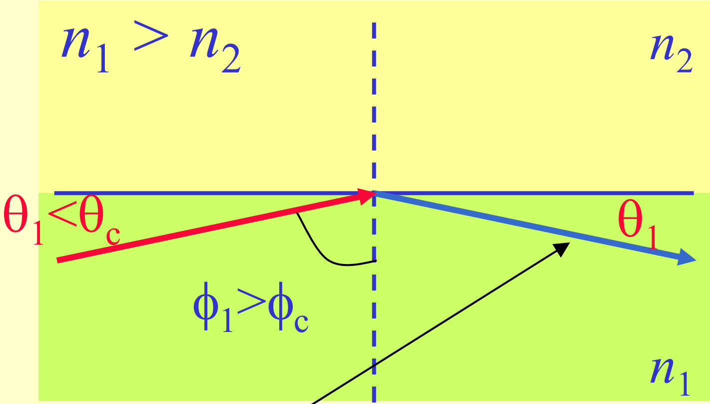
- **At ϕ_1 =** **Critical Angle ϕ_c**, **there is no reflection**

So, for ϕ_2 = **90°** (or $\theta_2 = 0°$)

$$n_1 \sin \phi_1 = n_2 \sin 90^o$$

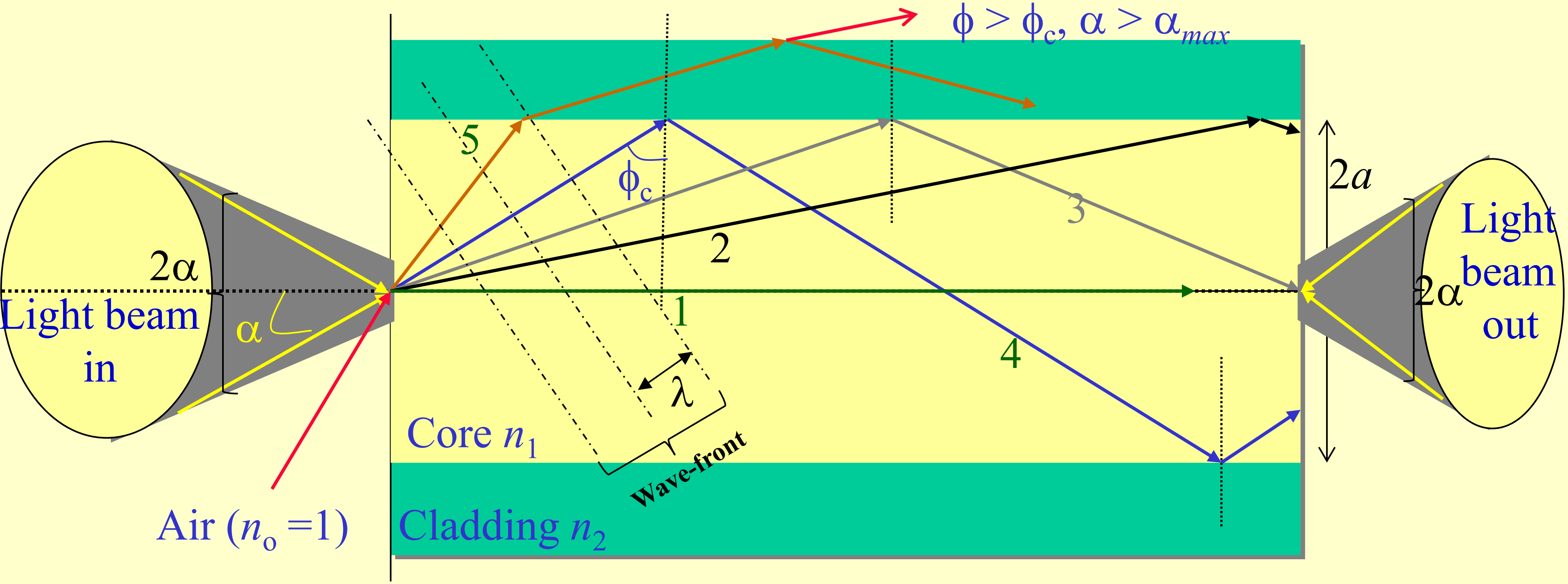
Thus, the critical angle $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$





Beyond the critical angle ($\phi_1 > \phi_c$), light ray becomes **totally internally reflected**

Ray Propagation in Fibre - *Bound Rays*



At high frequencies *f*, since

- *a* > light wavelength λ, light launched into the fibre core would propagate as plane TEM wave with $v_p = \omega/\beta_1$
- a < λ, light launched into the fibre will propagate within the cladding, thus unbounded and unguided plane TEM wave with $v_p = \omega/\beta_2$

Therefore, we have $n_2 k_0 = \beta_2 < \beta < \beta_1 = n_1 k_0$

Ray Propagation in Fibre - contd.

From Snell's Law: $n_0 \sin \alpha = n_1 \sin (90 - \phi)$

$\alpha = \alpha_{max}$ when $\phi = \phi_c$

Thus, $n_0 \sin \alpha_{max} = n_1 \cos \phi_c$

$\sin^2 x + \cos^2 x = 1$

Or $n_0 \sin \alpha_{max} = n_1 (1 - \sin^2 \phi_c)^{0.5}$

Since $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Then $$n_0 \sin \alpha_{\max} = n_1 \left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{0.5} = \left[n_1^2 - n_2^2\right]^{0.5}$$

$$= \left[n_1^2 - n_2^2\right]^{0.5} = Numerical\,Aperture(NA)$$

Therefore $n_0 \sin \alpha_{max} = \text{NA}$

Fibre acceptance angle $\alpha_{\max} = \sin^{-1}\left(\frac{NA}{n_0}\right)$

Ray Propagation in Fibre - contd.

Note $\frac{n_1 - n_2}{n_1} = \Delta$ Relative refractive index difference

Thus $$NA = n_1 (2\Delta)^{0.5}$$ $0.14 < NA < 1$

NA determines the light gathering capabilities of the fibre

Modes (Paths) in Fibre

- A fiber can support:
  - many modes (multi-mode fibre).
  - a single mode (single mode fibre).
- For the mode to propagate we must have

$$\underbrace{(4a\cos\varnothing_i)\frac{2\pi n_1}{\lambda_0} + 2\Delta\varnothing}_{\text{The total phase change}} = 2m\pi$$

where m is an integer

Or

$$\frac{4a\pi n_1\cos\varnothing_i}{\lambda_0} + \Delta\varnothing = m\pi$$

- For each value of m there will a corresponding value of ϕ_i, namely ϕ_m, that satisfies this equation.
- The dependence of $\Delta\phi$ at reflection on ϕ is such that we cannot obtain an explicit expression for ϕ in terms of m. **So the equation must be solved either numerically or graphically.**

Modes in Fibre

Let’s rewrite the equation as:

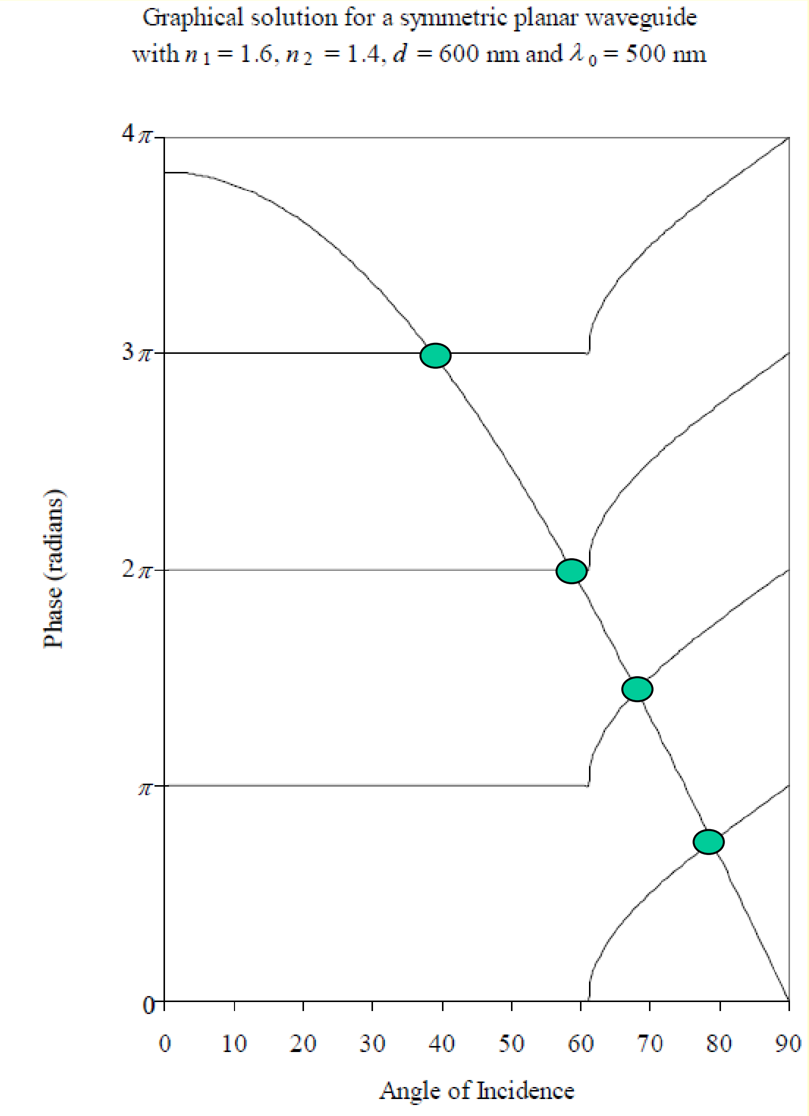
$$\frac{4a\pi n_1 cos\phi_i}{\lambda_0} = m\pi - \Delta\phi$$

The intersection of the two curve of

$$y = \frac{4a\pi n_1 cos\phi_i}{\lambda_0} \quad \text{and} \quad y = m\pi - \Delta\phi$$

defines the values of ϕ for which the modes propagates along the waveguide.

Note that, we are only interested in $\phi < \phi_c$.



Modes in Fibre

- Both TE_m and TM_m modes will be cut-off when

$$\frac{4a\pi n_1 cos\emptyset_i}{\lambda_0} = m\pi$$

Note, $cos\emptyset_c = (1 - sin^2\emptyset_c)^{0.5} = (1 - (^{n_2}/_{n_1})^2)^{0.5}$

$$\frac{4a\pi n_1}{\lambda_0}\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{0.5} = m\pi$$

Or

$$V = \frac{m\pi}{4}$$

***V* Parameter**

Modes in Fibre

- ***V* Parameter** - The number of modes [also known as the normalised frequency] supported in a fiber, which is determined by the **indices**, **operating wavelength** and the **diameter of the core**, given as:

$$V = \frac{a\pi n_1}{\lambda_0}\left[1 - \left(\frac{n_2}{n_1}\right)^2\right]^{0.5}$$

or

$$V = \frac{\pi a}{\lambda_0} NA$$

Note, a mode remains guided provided the following is satisfied

$$n_2 k_0 = \beta_2 < \beta < \beta_1 = n_1 \mathrm{k}_0$$

Modes in Fibre

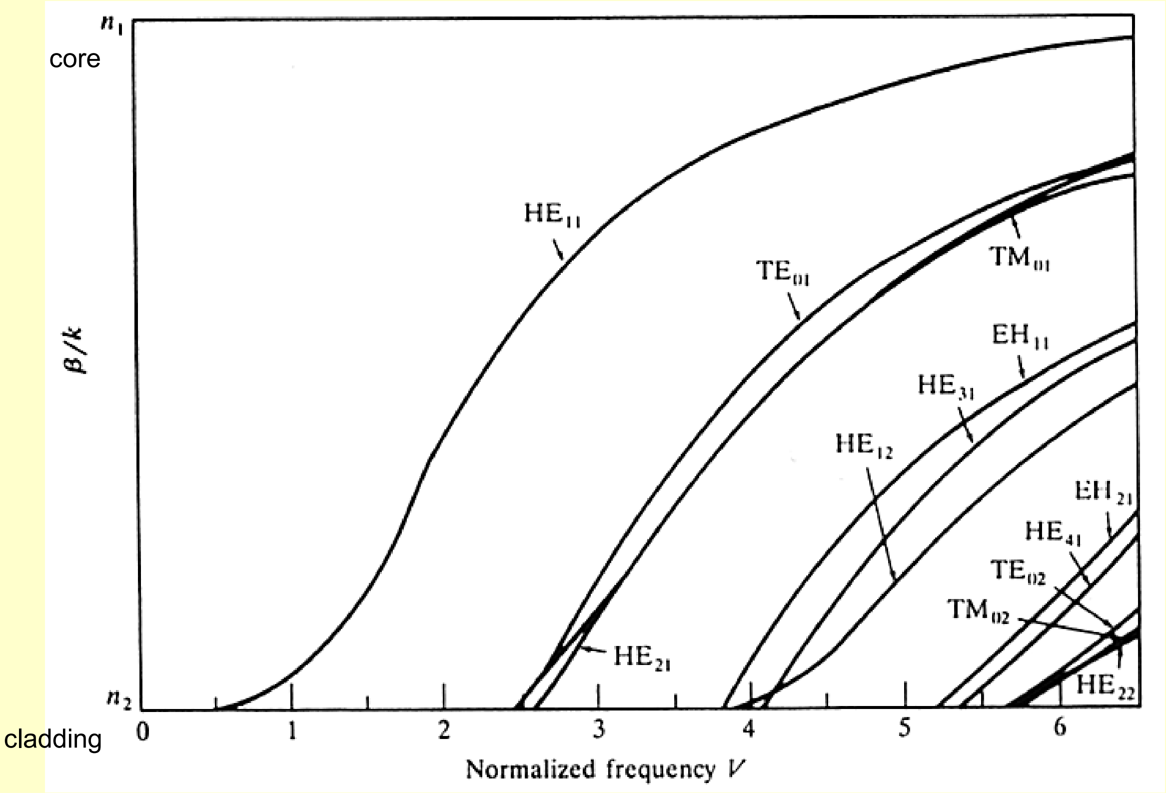
- The number of TE or TM modes propagating is given by:

$$N_m = 1 + INT(2V/\pi)$$

- For $V < \pi/2$, $N_m = 1$, i.e., only one mode ($m = 0$, the lowest order) will propagate. **In practice there will be two modes of TE_0 and TM_0.**

- **$V < 2.405$ - Corresponds to a single mode fiber.**
- By reducing the radius of the fiber, V goes down, and it becomes impossible to reach a point when only a single mode can be supported.

Modes in Fibre



$$n_2 k = k_2 \le \beta \le k_1 = n_1 k,$$
$$k = 2\pi / \lambda_0$$

$$\mathbf{E} \sim \exp(j\omega t - \beta z)$$

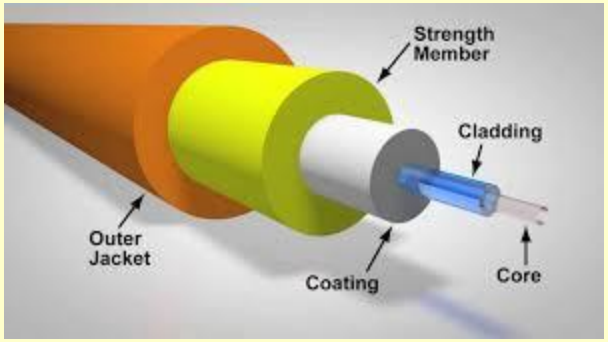
Modes in Fibre

Problem: Consider a symmetric planar dielectric waveguide of 100 um thick with core and cladding refractive indices of 1.54 and 1.5, respectively. Determine the number of possible modes at a operating wavelength of 1300 nm.

Solution:

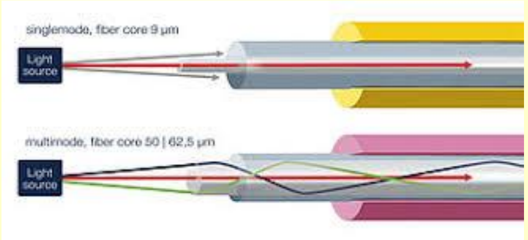
Basic Fibre Properties

- Cylindrical
- Dielectric
- Waveguide
- Low loss
- Usually fused silica
- Core refractive index > cladding refractive index
- Operation is based on total internal reflection



Types of Fibre

There are two main fibre types:



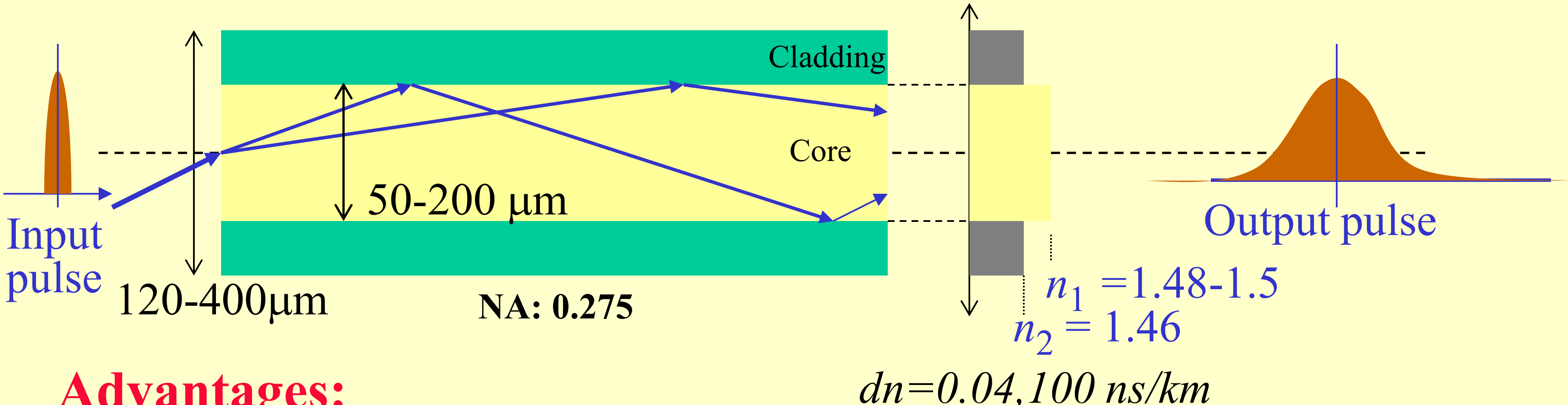
- **Step index:**
  - **Multi-mode**
  - **Single mode**
- **Graded index multi-mode**

Total number of guided modes M for multi-mode fibres:

Multi-mode SI $M = 0.5V^2$ Multi-mode GI $M \approx 0.25V^2$

SMSI $V < \mathbf{2.405}$

Step-index Multi-mode Fibre



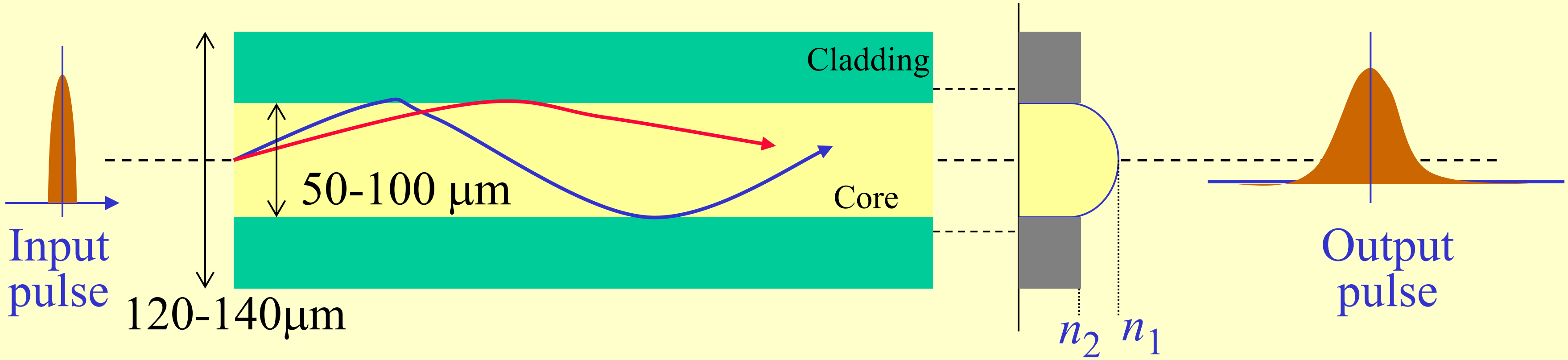
Advantages:

- Use of non-coherent optical light source, e.g., LED's
- Easy to connect similar fibres
- Lower tolerance requirements on fibre connectors.
- Cost effective

Disadvantages:

- Suffer from dispersion (i.e., low bandwidth (a few MHz)
- High power loss

Graded-index Multi-mode Fibre



$dn = 0.04$, 1ns/km

Advantages:

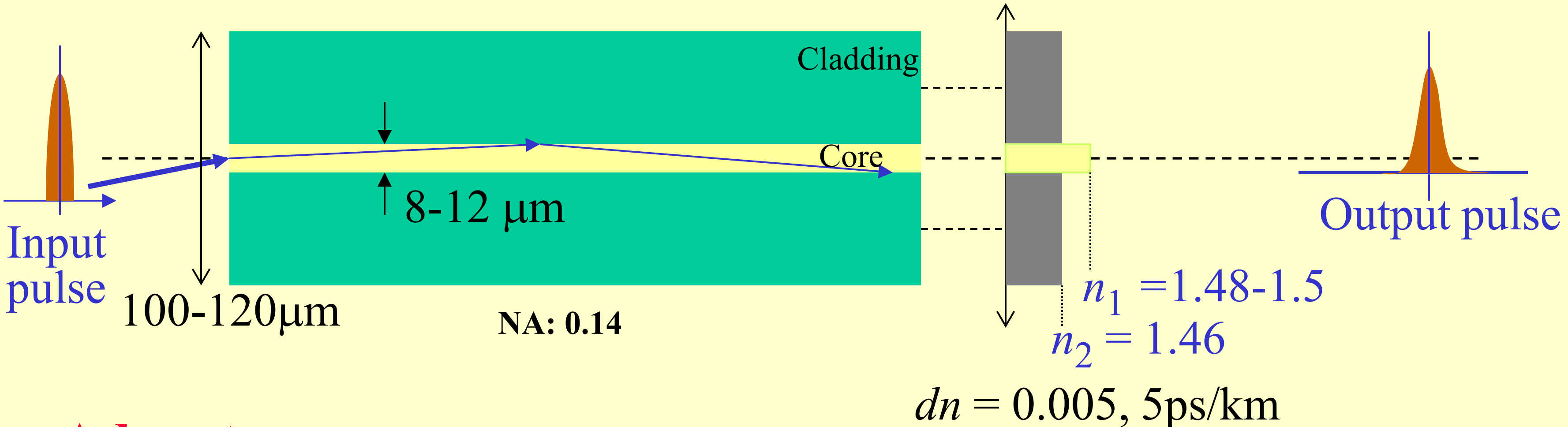
- Use of non-coherent optical light source, e.g. LED's
- Facilitates connecting similar fibres
- Imposes lower tolerance requirements on fibre connectors.
- Reduced dispersion compared with STMMF

Disadvantages:

- Lower bandwidth
- High power loss

Compared with STSMF

Step-index Single-mode Fibre



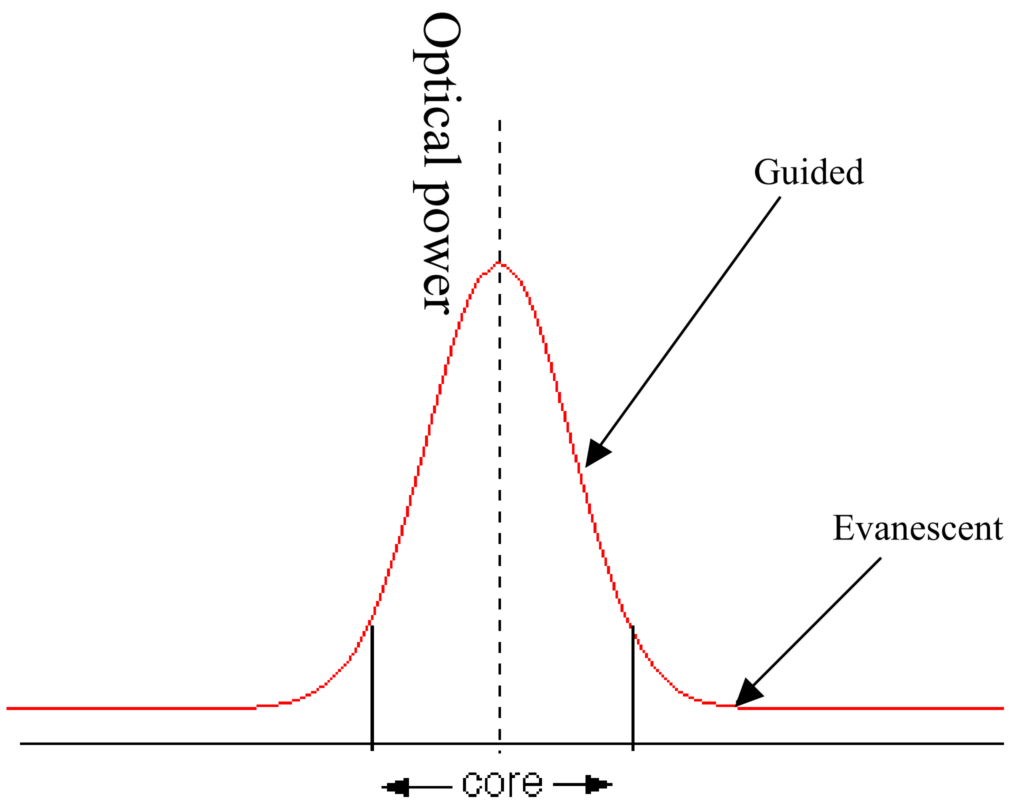
Advantages:

- Only one mode is allowed due to diffraction/interference effects.
- Use high power laser sources
- Facilitates fusion splicing similar fibres
- **Low dispersion, therefore high bandwidth (a few GHz)**.
- Low loss (0.1 dB/km)

Disadvantages:

- Cost

Single Mode Fiber - Power Distribution



Intensity profile of the fundamental mode

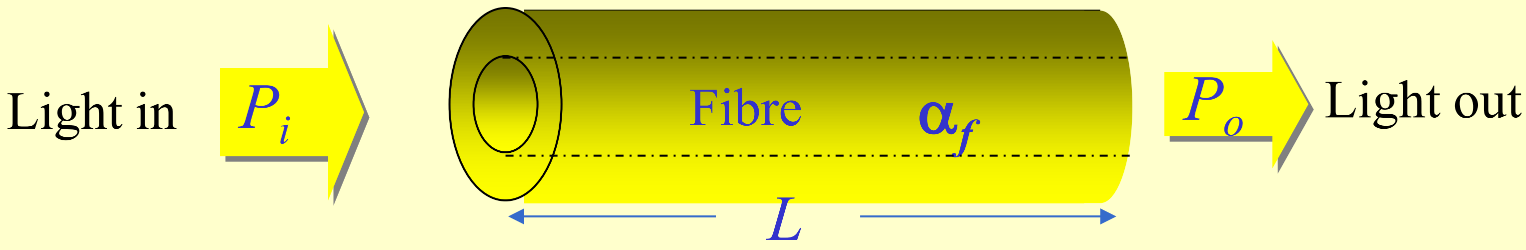
Fibre Characteristics

- The most important characteristics that limit the transmission capabilities are:

  - Attenuation (loss)

  - Dispersion (pulse spreading)

Attenuation (Loss) - *contd.*



The output power $P_o(L) = P_i(0).e^{-\alpha_f L}$

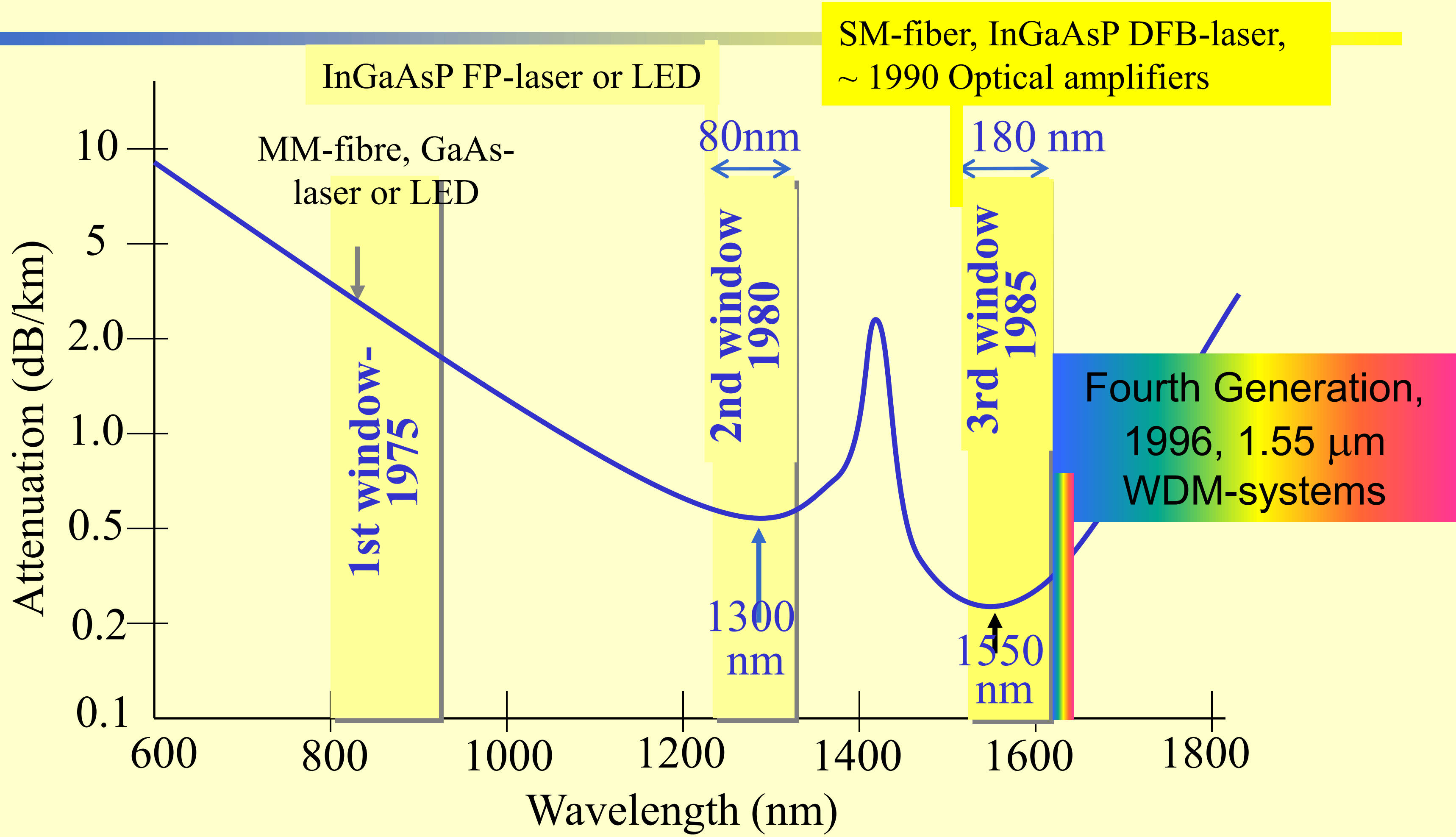
Fibre attenuation coefficient $\alpha_f = \frac{1}{L}\ln\left(\frac{P_o}{P_i}\right)$ km^{-1}

$(\alpha_f = \alpha_{scattering} + \alpha_{absorption} + \alpha_{bending})$

However, it is common to express fibre attenuation coefficient in dB/km:

$$\alpha_f = \frac{10}{L}\log\left(\frac{P_o}{P_i}\right) = 4.43\alpha_f \ \ (\text{km}^{-1})$$

Attenuation - Standard Fibre

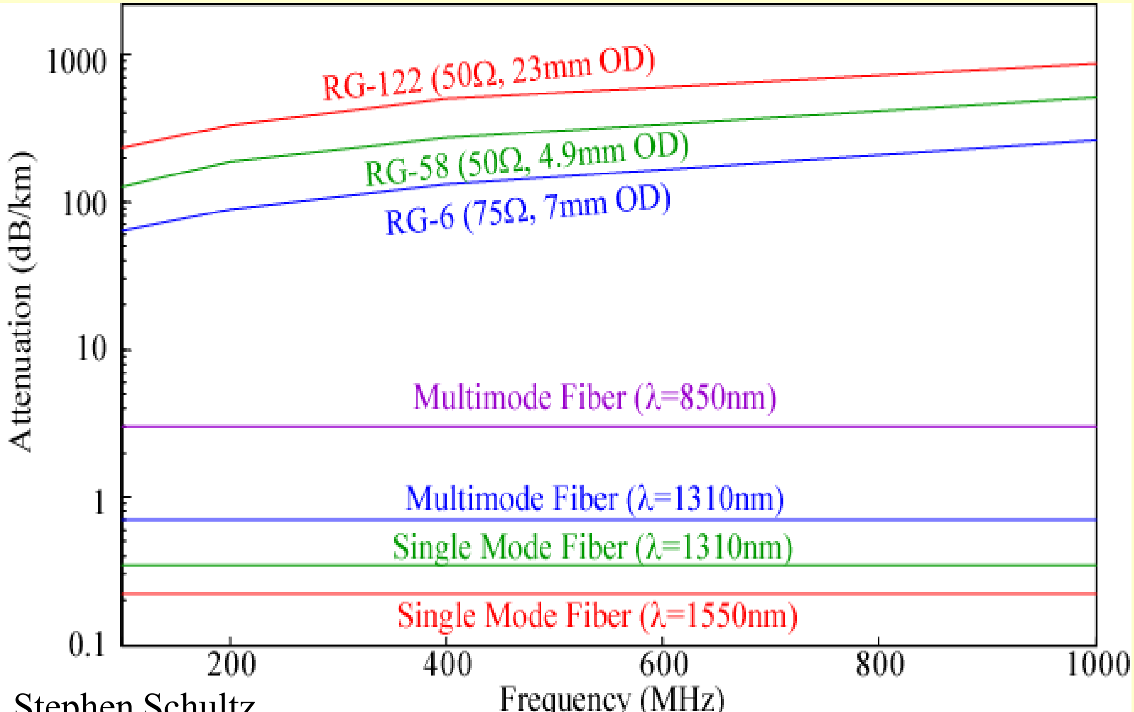


Bandwidth $\Delta f = \frac{c}{\lambda^2}\Delta\lambda$ $= 1.142 \times 10^{14}$ Hz $|_{\lambda 1300\ nm}$ + 2.2475 10^{14} Hz $|_{\lambda 1550\ nm}$

Fibre Attenuation - *contd.*

- In a typical system, the total loss could bas **20-30 dB**, before it needs amplification.

  So, at 0.2 dB/km, this corresponds to a distance of **100-150 km**.



Stephen Schultz

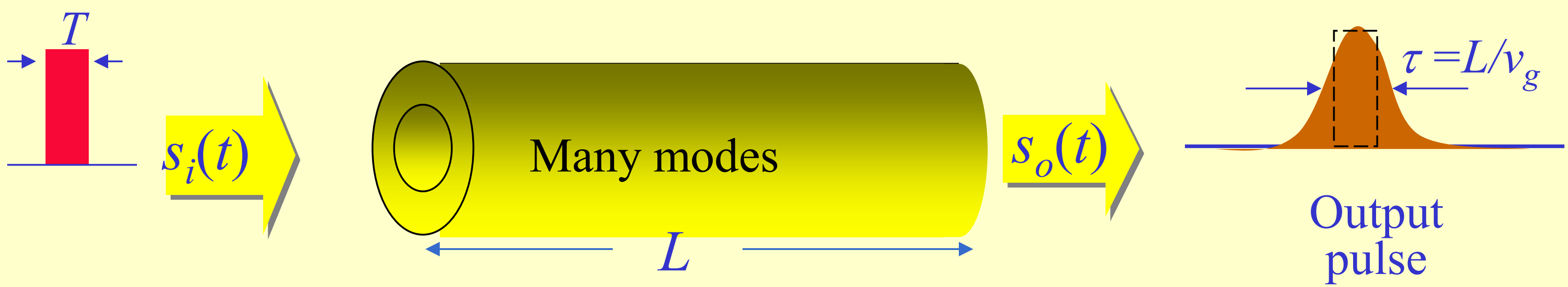
Attenuation along the fibre link can be measured using **Optical Time Domain Reflectometer**

Fibre Attenuation - *contd.*

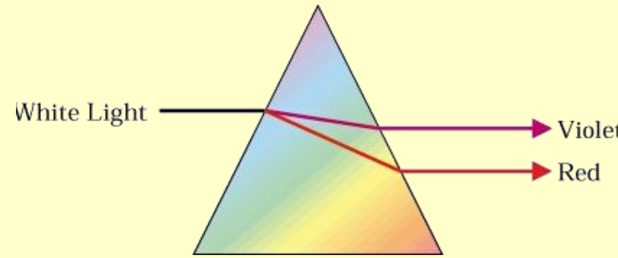
- Bending loss
  - Radiation loss at bends in the optical fiber
  - Insignificant unless R<1mm
  - Larger radius of curvature becomes more significant if there are accumulated bending losses over a long distance
- Coupling and splicing loss
  - Misalignment of core centers
  - Tilt
  - Air gaps
  - End face reflections
  - Mode mismatches

Fibre Dispersion

- Digital data carried composed of a large number of frequencies travelling at a given rate.
- There is a limit to the highest data rate (frequency) that can be sent down a fibre and be expected to emerge intact at the output.
    - Because of a phenomenon known as **Dispersion** (pulse spreading), which limits the **"Bandwidth"** of the fibre.



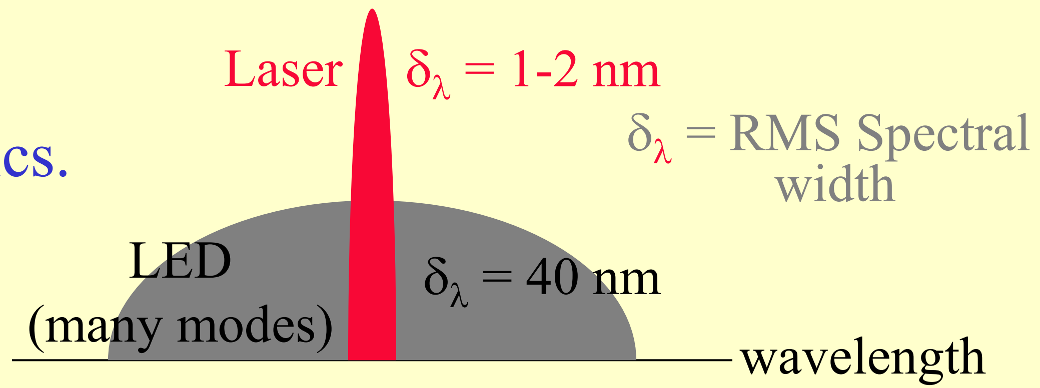
Cause of Dispersion:
- Chromatic (intramodal) dispersion
- Modal (intermodal) dispersion



Chromatic (Intramodal) Dispersion

- Since $v_g = f(\lambda)$, then in any given mode, different spectral components of a pulse is traveling through the fibre at different speed.

- It depends on the light source spectral characteristics.



- May occur in all fibre, **but is the dominant in single mode fibre.**

- **Main causes:**
  - Material dispersion - different wavelengths => different speeds
  - Waveguide dispersion: different wavelengths => different angles

CD - Material Dispersion

- In intensity modulated light sources, all modes are excites equally at the input to the fibre:
  - Each mode:
    - carry the same amount of information
    - contains all the spectral components in the fibre
  - Each spectral component@
    - is modulated in the same way
    - travel independently and undergo a time delay (or a group delay) per unit length as it propagates along the fibre.

$$v_g = \frac{d\omega}{d\beta} = \frac{d(2\pi f)}{d(\frac{2\pi}{\lambda})} = \frac{d\omega}{df} \times \frac{df}{d\lambda} \times \frac{d\lambda}{d\beta}$$

$$v_g = 2\pi \times \frac{df}{d\lambda} \times \left(-\frac{\lambda^2}{2\pi}\right) = -\lambda^2 \frac{df}{d\lambda}$$

Note: the vaccume wavelength $\lambda_0 = n\,\lambda$

Material Dispersion

So we can re-write the group velocity as:

$$v_g = -\frac{\lambda_0^2}{n^2}\left(\frac{df}{d\lambda_0} \times \frac{d\lambda_0}{d\lambda}\right) = -\frac{\lambda_0^2}{n^2}\left(\frac{c}{\lambda_0^2} \div \frac{d\lambda}{d\lambda_0}\right)$$

$$v_g = \frac{c}{n^2}\left(\frac{n^2}{n - \lambda_0 dn/\lambda_0}\right) = \left(\frac{c}{n - \lambda_0 dn/\lambda_0}\right)$$

Given that the group index $n_g = \frac{c}{v_g}$.

Then we have

Due to material dispersion

$$n_g = n - \lambda_0 \frac{dn}{d\lambda_0}$$

The group delay over a transmission span of L is: $\tau_g = \frac{L}{v_g} = \frac{L}{c}\left(n - \lambda_0 \frac{dn}{d\lambda_0}\right)$

Material Dispersion

Note that $\tau_{mat} = \delta\lambda_0 \dfrac{d\tau_g}{d\lambda_0}$

If the wavelength spread is $\delta\lambda_0$, the we have RMS pulse broadening

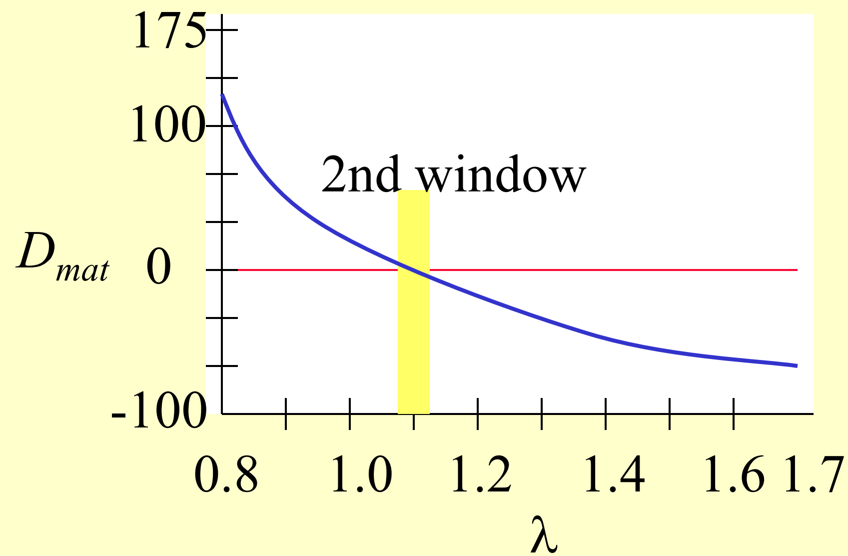
Most important

$$\tau_{mat} = -L\delta\lambda_0 \frac{\lambda_0}{c}\left|\frac{d^2n}{d\lambda^2{}_0}\right| \quad ns/km$$

Where material dispersion coefficient:

$$D_{mat} = \frac{d}{d_\lambda}\left(\frac{1}{v_g}\right)$$

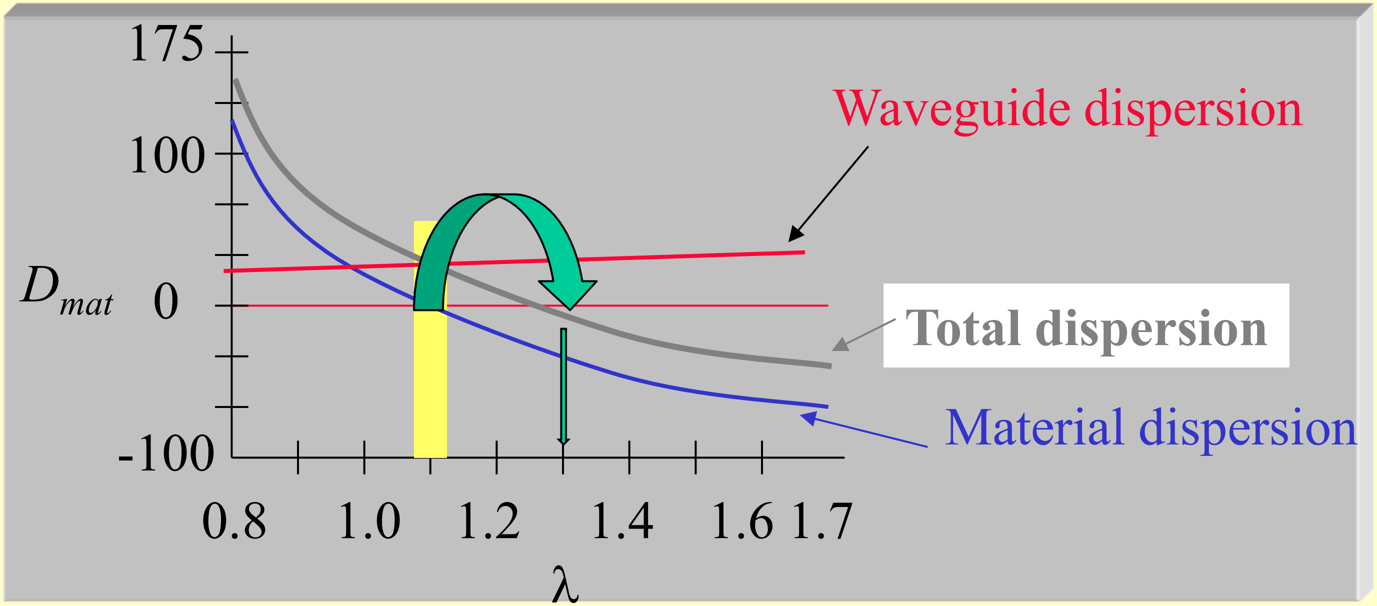
$$D_{mat} = -\frac{\lambda_0}{c}\frac{d^2n}{d\lambda_0{}^2} \quad ps/nm.km$$



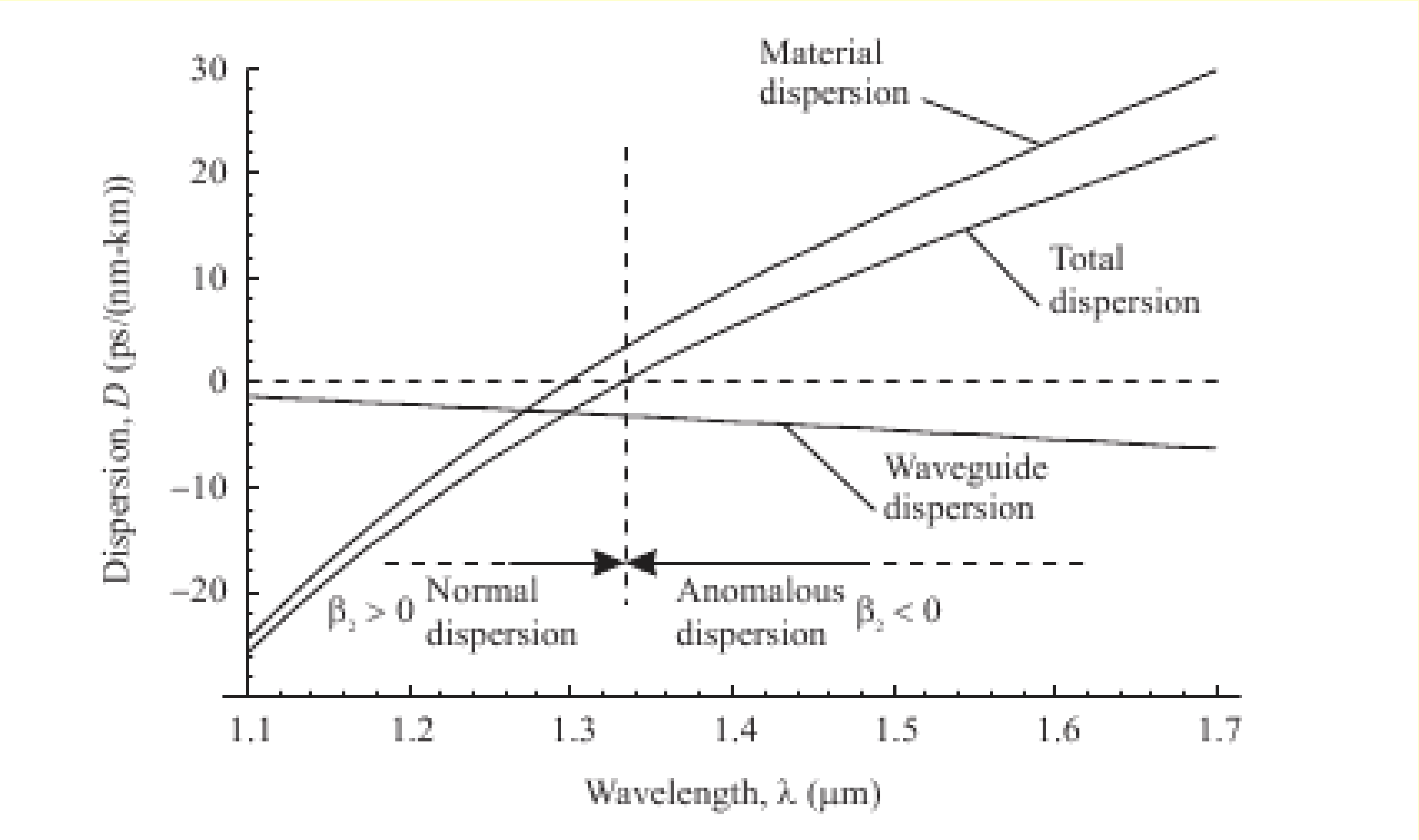
Note: Negative sign, indicates that low wavelength components arrives before higher wavelength components.

CD - Waveguide Dispersion

- This results from variation of the group velocity with wavelength for a particular mode. Depends on the size of the fibre.
- This can usually be ignored in multimode fibres, since it is very small compared with material dispersion.
- However it is significant in monomode fibres.



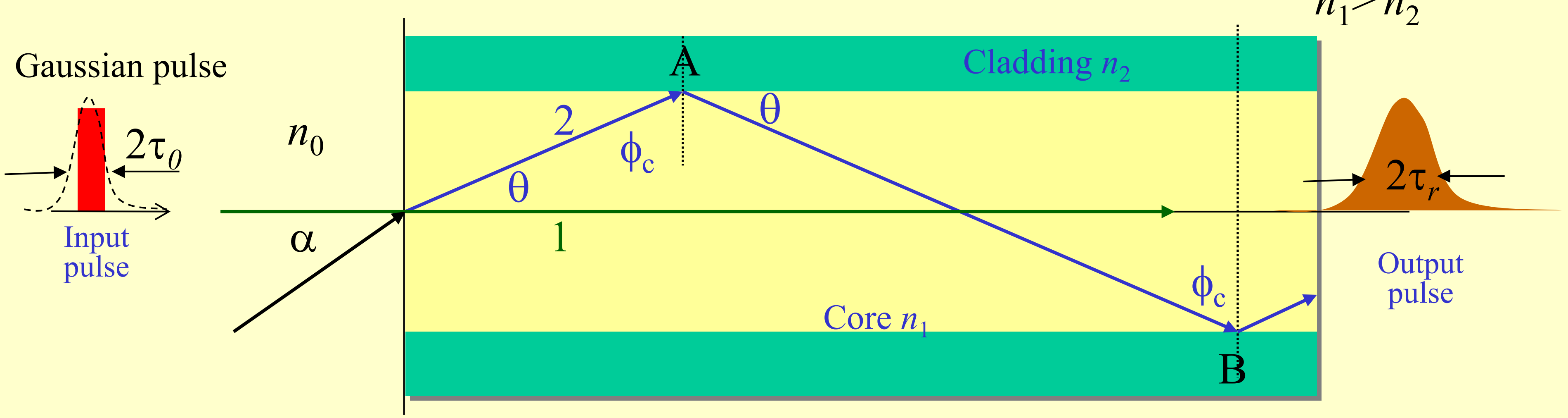
Chromatic (Intramodal) Dispersion



Modal (Intermodal) Dispersion

- Lower order modes travelling almost parallel to the centre line of the fibre cover the **shortest distance**, thus reaching the end of fibre sooner.
- The higher order modes (more zig-zag rays) take a **longer route** as they pass along the fibre and so reach the end of the fibre later.

- Mainly in multimode fibres



Modal Dispersion - SIMMF

The Gaussian pulse at the transmitter:

$$P(t, z=0) = P_0 \exp\left[-\frac{2(t-t_0)^2}{\tau_0^2}\right]$$

The pulse at a distance z is:

$$P(t, z) = \frac{\tau_0}{\tau_r} P_0 \exp\left[-\frac{2(t-t_z)^2}{\tau_r^2}\right]$$

Now

$$\tau_r^2 = \tau_0^2 + \delta\tau^2$$

$\delta\tau^2$ → Pulse broadening

Next, let's look at pulse broadening.

Modal Dispersion - SIMMF

The time taken for ray 1 to propagate a length of fibre L gives the minimum delay time:

$$t_{\min} = \frac{Ln_1}{c}$$

The time taken for the ray 2 to propagate a length of fibre L gives the maximum delay time:

$$t_{\max} = \frac{L/\cos\theta}{c/n_1}$$

Since $\sin\phi_c = \frac{n_2}{n_1} = \cos\theta$

The delay difference

$$\delta T_s = \delta\tau = t_{\max} - t_{\min} = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right)$$

Note, the relation ship between transmission bandwidth B_T and pulse broadening

$$\delta\tau < T_b \qquad where\ \ T_b \approx \frac{1}{B_T}$$

Therefore

$$\delta\tau B_T < 1 \Rightarrow B_T \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right) < 1 \Rightarrow B_T L < \left(\frac{n_2 c}{n_1(n_1 - n_2)}\right)$$

Modal Dispersion - SIMMF

$$B_T L < \left(\frac{n_2 c}{n_1 (n_1 - n_2)} \right)$$

So, for a given fibre with fixed n_1 and n_2, Bandwidth Distance Product $\boldsymbol{B_T L}$ **is constant**

So, what is the implication of this?

Lets' consider two scenarios:

S1: $n_1 = 1.5$, and $n_2 = 1$ (i.e., un-cladded fibre)

$B_T L$ < 0.4 Gb/s-m or 0.4 Mb/s-km

S2: $n_1 = 1.5$, and $\Delta = 1\%$ (cladded fibre)

$B_T L$ < 20 Gb/s-m or 1 Mb/s-20 km

So, small is Δ, the higher would be the data rate! ***One more reason not to use un-cladded***

Modal Dispersion - SIMMF

For $\Delta << 1$, $\Delta = \frac{(n_1 - n_2)}{n_2}$ and $NA = n_1 (2\Delta)^{0.5}$

The delay difference $\delta T_s \approx \frac{L n_1^{\,2}}{c n_2} \Delta$ $\delta\tau = \delta T_s \approx \frac{L(NA)^2}{2 c n_1}$

For a rectangular input pulse, the RMS pulse broadening due to modal dispersion at the output of the fibre is:

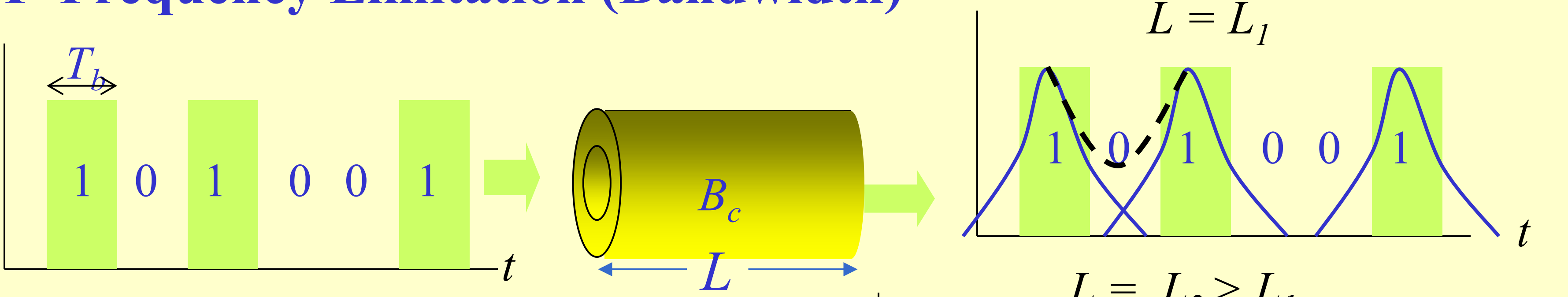
$$\tau_{\text{modal}} = \frac{L n_1 \Delta}{3.5c} = \frac{L(NA)^2}{7 n_1 C}$$

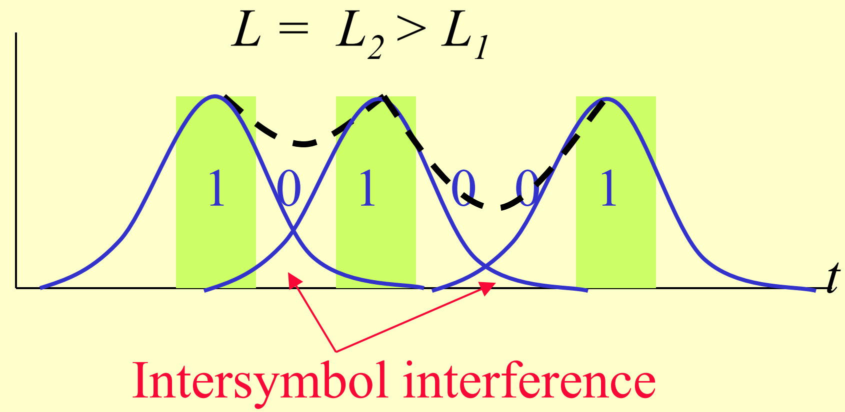
Total dispersion = Chromatic dispersion + Modal dispersion

$$\tau_T = [\tau_{chrom}{}^2 + \tau_{modal}{}^2]^{1/2}$$

Dispersion - Consequences

I- Frequency Limitation (Bandwidth)





- **Maximum frequency limitation** of signal, which can be sent along a fibre
- **Intersymbol interference (ISI)**, which is unacceptable in digital systems which depend on the precise sequence of pulses.

II- Distance: A given length of fibre, has a maximum frequency (bandwidth), which can be sent along it. To increase the bandwidth for the same type of fibre one needs to decrease the length of the fibre.

Bandwidth Limitations

- Maximum channel bandwidth B_c:
  - For non-return-to-zero (NRZ) data format: $B_c = B_T/2$
  - For return-to-zero (RZ) data format: $B_c = B_T$

- For zero pulse overlap at the output of the fibre $\boldsymbol{B_T <= 1/(2\tau_r)}$
  where τ_r is the pulse width.
  For MMSF: $\boldsymbol{B_T \text{ (max)} = 1/2\delta T_s}$

- For a Gaussian shape pulse: $\boldsymbol{B_T \leq 0.2/\tau}_{rms}$
  where τ_{rms} is the RMS pulse width.
  For MMSF: $\boldsymbol{B_T \text{ (max)} = 0.2/\tau}_{\text{modal}}$
  or
  $\boldsymbol{B_T \text{ (max)} = 0.2/\tau}_T$ Total dispersion

Bandwidth Distance Product (BDP)

The BDP is the bandwidth of a kilometer of fibre, and is a constant for any particular type of fibre.

$$B_{opt} * L = B_T * L \qquad \text{(MHz-km)}$$

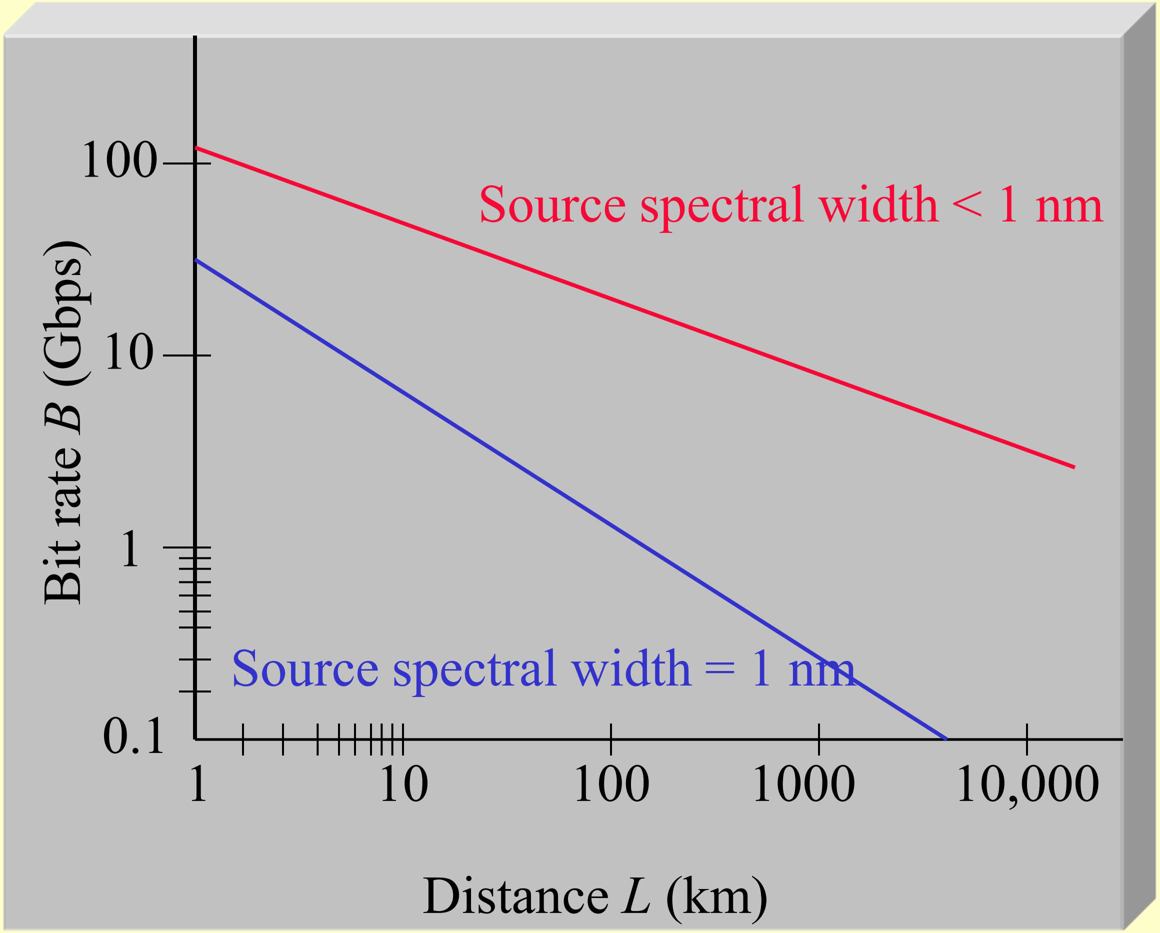
For example, A multimode fibre has a BDP of 20 MHz.km, then:-

- 1 km of the fibre would have a bandwidth of 20 MHz
- 2 km of the fibre would have a bandwidth of 10 MHz

Typical BDP for different types of fibres are:

- Multimode: 6 - 25 MHz.km
- Single Mode: 500 - 1500 MHz.km
- Graded Index: 100 - 1000 MHZ.km

Bandwidth Distance Product



D_{mat} = 17 ps/km.nm

Controlling Modal Dispersion in SIMMF

Use Graded index MMF

- Shortest path with the lowest speed and longest path with higher speed (lower n_{core}).

Then $$\delta\tau = \frac{n_2 L}{2c}\left(\frac{n_1}{n_2} - 1\right)^2 \quad \text{and} \quad \delta T_s \approx \frac{L n_1 \Delta^2}{2c}$$

For n_2 1.45 and $\Delta = \frac{(n_1 - n_2)}{n_2} = 0.01$, $\delta\tau = 0.25$ ns/km

For SIMMF $\delta\tau = 50$ ns/km

- The RMS pulse broadening $$\tau_{modal-GI} = \frac{L n_1 \Delta^2}{34.6C}$$

Controlling Dispersion

For single mode fibre:

- Wavelength 1300:
  - Dispersion is very small
  - Loss is high compared to 1550 nm wavelength
- Wavelength 1550:
  - Dispersion is high compared with 1300 nm
  - Loss is low

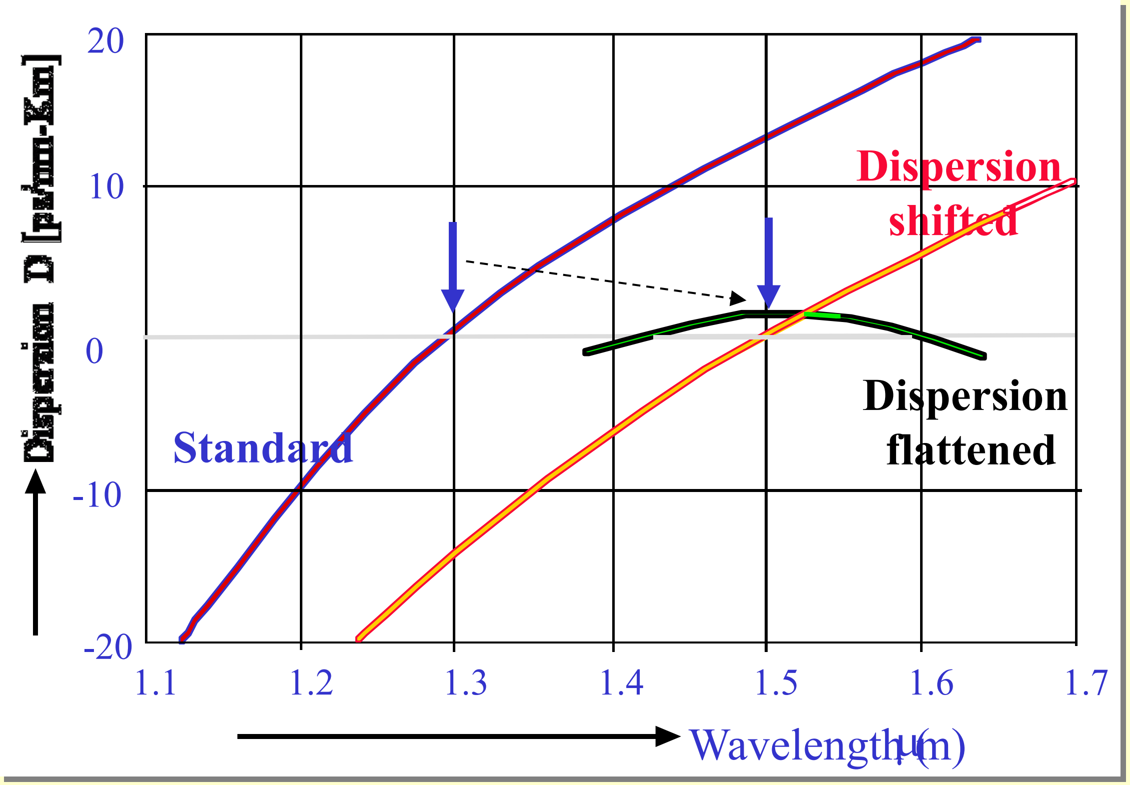
Limitation due to dispersion can be removed by moveing zero-diepersion point from 1300 nm to 1550 nm. *How?*

By controlling the waveguide dispersion.

Fibre with this property are called Dispersion-Shifted Fibres

Controlling Dispersion

Dispersion-Shifted Fibre



Basic idea

- Change the refractive index profile in cladding and core
- Thus introducing negative dispersion

References

- http://www.gatewayforindia.com/technology/opticalfiber.htm
- Senior: http://www.members.tripod.com/optic1999/

Summary

- Nature of light : Particle and wave
- Light is part of EM spectrum
- Phase and group velocities
- Reflection, refraction and total internal reflection etc.
- Types of fibre
- Attenuation and dispersion